Toulouse School of Economics, 2009-2010
Macroeconomics II – Franck Portier

# Homework 2

## Problem I – A Simplified Real-Business-Cycle Model with Additive Technology Shocks

Consider an economy consisting of a constant population of infinitely-lived individuals. The representative individual maximizes the expected value of $\sum_{t=0}^{\infty} u(C_t)/(1+\rho)^t$, $\rho > 0$. The instantaneous utility function,$u(C_t)$, is $u(C_t) = C_t - \theta C_t^2$, $\theta > 0$. Assume that $C$ is always in the range where $u'(C)$ is positive.

Output is linear in capital, plus an additive disturbance: $Y_t = AK_t + e_t$. There is no depreciation; thus $K_{t+1} = K_t + Y_t - C_t$, and the interest rate is $A$. Assume $A = \rho$. Finally, the disturbance follows a first-order autoregressive process: $e_t = \phi e_{t-1} + \varepsilon_t$, where $-1 < \phi < 1$ and where the $\varepsilon_t$'s are mean-zero, i.i.d. shocks.

**1** – Find the first-order condition (Euler equation) relating $C_t$ and expectations of $C_{t+1}$.

**2** – Guess that consumption takes the form $C_t = \alpha + \beta K_t + \gamma e_t$. Given this guess, what is $K_{t+1}$ as a function of $K_t$ and $e_t$?

**3** – What values must the parameters $\alpha$, $\beta$, and $\gamma$ have for the first-order condition in part (1) to be satisfied for all values of $K_t$ and $e_t$?

**4** – What are the effects of a one-time shock to $\varepsilon$ on the paths of $Y$, $K$, and $C$?

## Problem II – An analytic model with log-linear depreciation

Consider a model economy populated with a representative household and a representative firm. The firm has a Cobb-Douglas technology:

$$Y_t = Z_t K_t^{\gamma} N_t^{1-\gamma} \tag{1}$$

where $K_t$ is capital, $N_t$ labor input, and $Z_t$ a stochastic technological shock. All profits of the firm are distributed to the household. Capital evolves according to the log linear relation

$$K_{t+1} = A K_t^{1-\delta} I_t^{\delta} \qquad 0 < \delta \leq 1 \tag{2}$$

where $\delta$ is the rate of depreciation and where $I_t$ is investment in period $t$.

The representative household works $N_t$, consumes $C_t$ in period $t$, and ends the period with a quantity of money $M_t$.

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log C_t + \omega \log \frac{M_t}{P_t} - V(N_t) \right] \tag{3}$$

where $V$ is a convex function. At the beginning of period $t$ there is a stochastic multiplicative monetary shock denoted by $\mu_t$. The money holdings $M_{t-1}$ carried from the previous period are multiplied by $\mu_t$, so that the household starts period $t$ with money holdings $\mu_t M_{t-1}$. Capital is accumulated by the household and rented to the firm.

Let $\kappa$ denote the real rental rate of capital, $P$ the price of the final good and $W$ the nominal wage.

We first assume that $\delta = 1$ (full depreciation).

**1** – Write down the budget constraint of the household and the profit function of the firm

**2** – Derive FOCs of the utility and profit maximization

**3** – Define a competitive equilibrium of this economy

**4** – Solve the model and show that $N_t$ is constant along an equilibrium path.

**5** – Derive a $AR(1)$ process for log of output. Draw the Impulse Response Function to a unit shock to $z$ and $\mu$, assuming that both shocks are *iid*. What are the determinants of the' size and persistence of those IRFs?

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We assume now that $\delta \in ]0,1[$.

**6** – What is the economic meaning of equation (2)?

**7** – We still denote $\kappa_t$ the real return in $t$ on investment $I_{t-1}$:

$$\kappa_t = \frac{\partial Y_t}{\partial I_{t-1}} \tag{4}$$

and note that the price, in terms of current output, of capital to be transmitted to the next period is not one as when $\delta = 1$. We denote it as $q_t$ (like Tobin's $q$), and it is equal to:

$$q_t = \frac{\partial K_{t+1}/\partial K_t}{\partial K_{t+1}/\partial I_t} \tag{5}$$

**8** – Comment the new budget constraint of the household

$$C_t + \frac{M_t}{P_t} + I_t = \frac{W_t}{P_t}N_t + \kappa_t(I_{t-1} + q_{t-1}K_{t-1}) + \frac{\mu_t M_{t-1}}{P_t} \tag{6}$$

**9** – Solve for the competitive equilibrium of this economy and compute $\frac{I_t}{C_t}$ and $\frac{M_t}{P_t Y_t}$

**10** – Compute the constant level of $N$.

**11** – Derive the new process for log of output. Draw typical IRFs and discuss of their shape as a function of parameters.