Toulouse School of Economics, 2009-2010
Macroeconomics II – Franck Portier

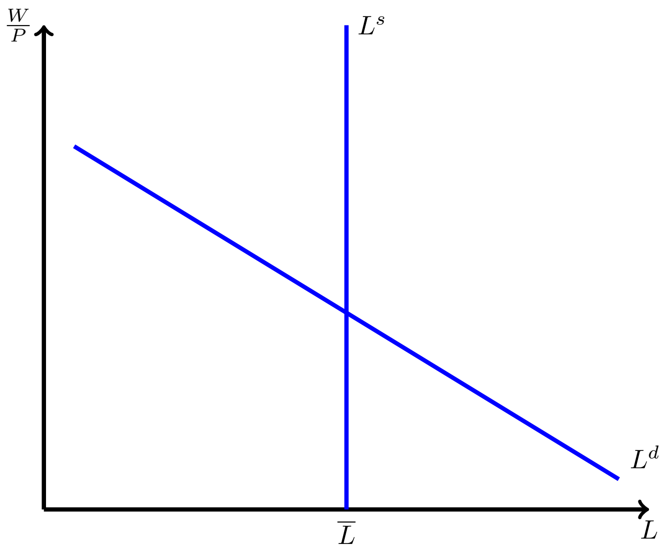
Homework 1
Solutions

Problem I – An AD-AS Model

1 – $\max \Pi = PAL^\alpha - WL \rightsquigarrow$ FOC: $L^d = \left(\frac{1}{\alpha A}\frac{W}{P}\right)^{1/-\alpha-1)}$

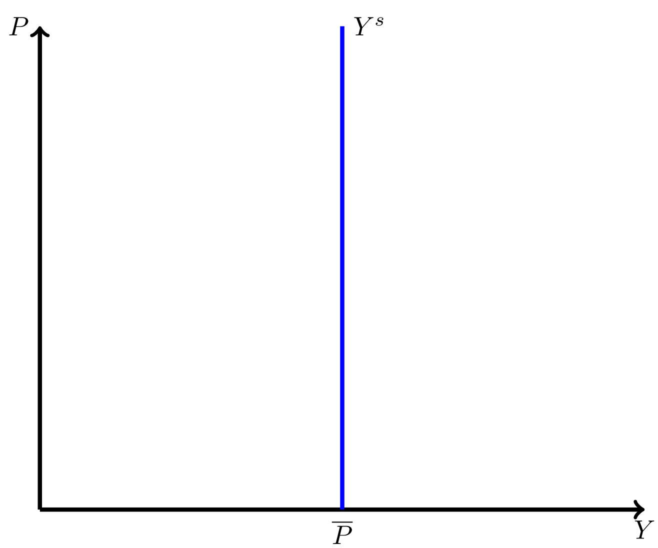
2 – Equilibrium on the labor market: $L^d = L^s \rightsquigarrow \frac{W}{P} = \alpha A\overline{L}^{\alpha-1}$ and $Y = \overline{Y} = A\overline{L}^\alpha$

Figure 1: Equilibrium of the labor market



3 – See figure 2.

Figure 2: Aggregate Supply



4 – IS: set of (Y, i) such that planned and actual expenditures are equal, *i.e.* $Y = C + I + G$, which gives

$$Y = \frac{1}{1-c(1-\tau)} - \frac{\gamma}{1-c(1-\tau)}\ i \tag{IS}$$

5 – LM: set of (Y, i) such that the money market clears; *i.e.* $M^d = M^s \rightsquigarrow$

$$i = -\frac{1}{\beta}\frac{\overline{M}}{P} + \frac{\alpha}{\beta}Y \tag{LM}$$

6 – IS-LM :

$$Y^D = \frac{1}{1-(1-\tau)c + \frac{\alpha\gamma}{\beta}}(G_c T + \overline{C} + \overline{I}) + \frac{\frac{\gamma}{\beta}}{1-(1-\tau)c + \frac{\alpha\gamma}{\beta}}\frac{\overline{M}}{P}$$

We can check that $\frac{\partial Y^D}{\partial P} < 0$

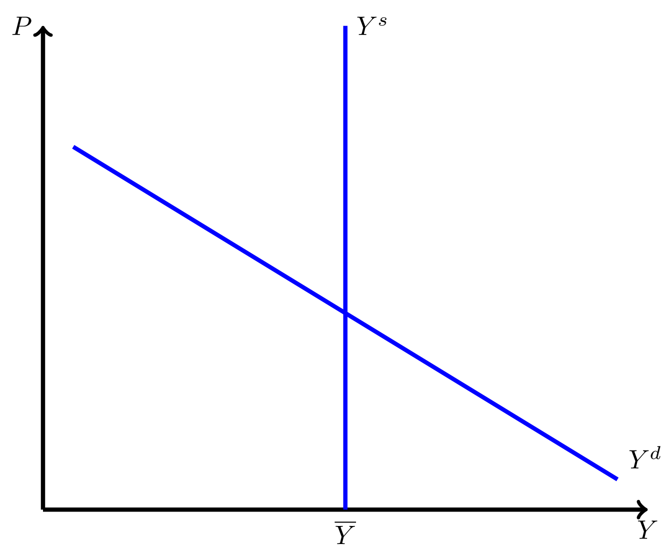
7 –

$$G = T + \tau Y + \frac{\overline{M}}{P} + \frac{B}{P}$$

where B/P is public debt issuing.
• possible policies are
– Fiscal policy $dG = dB/P$, $dG = dT$, $dG = dM/P$
– Tax cut: dT = -d B/P
– Open market $dM/P = dB/P$

8 – See figure 3

Figure 3: Aggregate Demand- Aggregate Supply



9 – Equilibrium real wage: $\frac{\overline{W}}{P} = \alpha A \overline{L}^{\alpha-1} \rightsquigarrow$ labor market equilibrium price : $\overline{P} = (\alpha A)^{-1}\overline{L}^{1-\alpha}\overline{W}$

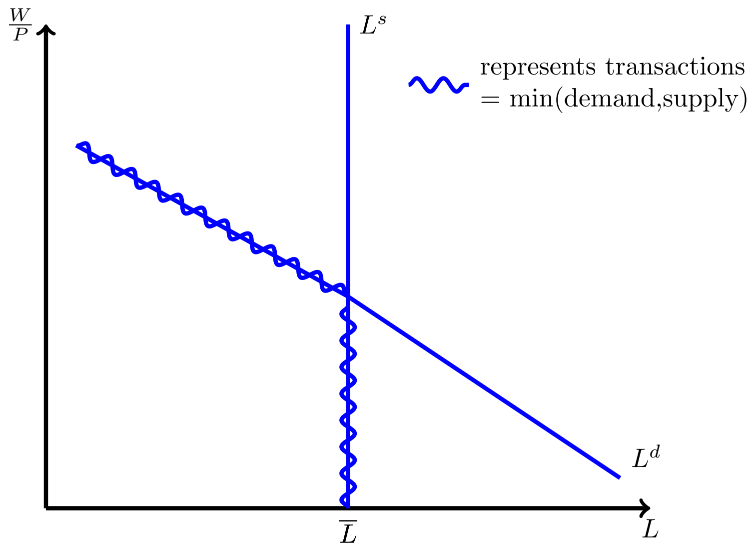
This price needs not to coincide with the price that is given by aggregate demand $\rightsquigarrow$ there can be unemployment:
– if $P < \overline{P}$, $L^d < \overline{L} \rightsquigarrow$ unemployment
– if $P > \overline{P}$, $L^d > \overline{L} \rightsquigarrow$ underemployment

We assume that transactions are given by the min of demand and supply (voluntary exchange assumption) $\rightsquigarrow$ see figure 4.

10 – The AD curve is the same than before. The AS curve is now given by:

$$L = \begin{cases} L^d & \text{if } P < \overline{P} \\ \overline{L} & \text{if } P \geq \overline{P} \end{cases} \quad \Leftrightarrow \quad L = \begin{cases} \left(\frac{\overline{W}}{\alpha A}\right)^{1/(\alpha-1)} P^{1/(1-\alpha)} & \text{if } P < \overline{P} \\ \overline{L} & \text{if } P \geq \overline{P} \end{cases}$$

Figure 4: Transactions on the labor market

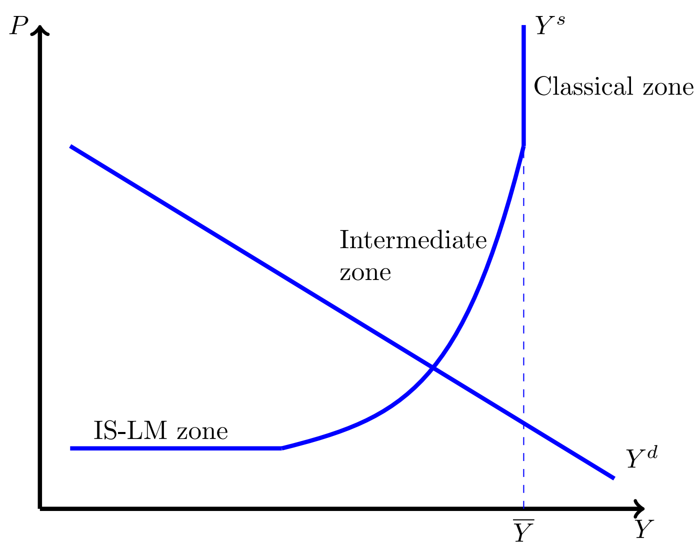


and therefore

$$Y^S = \begin{cases} A\left(\frac{\overline{W}}{\alpha A}\right)^{\alpha/(\alpha-1)} P^{\alpha/(1-\alpha)} & \text{if} \quad P < \overline{P} \\ \overline{Y} & \text{if} \quad P \geq \overline{P} \end{cases}$$

11 – See figure 5

Figure 5: AD-AS Model



Problem II – A Lucas 73 type model

See Romer's book, chapter 6, part A

$$\log(M_t/P_t) = \alpha_0 + \alpha_1 \log y_t + \alpha_2 R_t + u_t \tag{1}$$

1- A priori, the signs of the coefficients are: $\alpha_1 > 0$ and $\alpha_2 < 0$. A high activity (high y) implies a need for liquidity and then a high demand of money. The nominal interest rate represents the opportunity cost to hold money, thus if R is high, the demand of money is low.

2- $R_t = r_t + \pi_t$, $y_t = y$, $r_t = r$ then (1) is equivalent to

$$\begin{aligned} m_t - p_t &= (\alpha_0 + \alpha_1 \log y_t + \alpha_2 r_t) + \alpha_2 \pi_t + u_t \\ &= \gamma + \alpha \pi_t + u_t \end{aligned} \tag{2}$$

where $\gamma = \alpha_0 + \alpha_1 \log y_t + \alpha_2 r_t$ and $\alpha = \alpha_2$

3- We can observe that when π is high, $m - p$ is low. This is in accordance with the money demand if $\alpha < 0$, if one accumulates expected and actual inflation.

4- The problem with equation (9) is that we do not observe expectations. We only observe actual inflation, and we don't know how much was expected.

5- Adaptive expectations (error correcting mechanism). The anticipations of inflation are proportional to the last period prediction errors.

$$\pi_t - \pi_{t-1} = \lambda(\Delta p_t - \pi_{t-1}) \tag{3}$$

6- (3) implies

$$\begin{aligned} \pi_t &= \lambda \Delta p_t + (1-\lambda)\pi_{t-1} \\ &= \lambda \Delta p_t + (1-\lambda)\{\lambda \Delta p_{t-1} + (1-\lambda)\pi_{t-2}\} \\ &= \lambda \Delta p_t + \lambda(1-\lambda)\Delta p_{t-1} + (1-\lambda)^2 \pi_{t-2} \\ &= \lambda \textstyle\sum_{i=0}^{\infty} \delta p_t - i \end{aligned} \tag{4}$$

if we assume $\lim_{j\to\infty}(1-\lambda)^j \pi_{t-j} = 0$. It is a weighted average of past observation. The parameter λ represents the weight of past observations in the expectations of inflation.

7- We have

$$m_t - p_t = \gamma + \alpha \pi_t + u_t \tag{5}$$

and

$$\pi_t = \lambda \Delta p_t + (1-\lambda)\pi_{t-1}$$

Then

$$\begin{aligned} \pi_t &= \lambda \Delta p_t + (1-\lambda)\{\frac{(m_{t-1} - p_{t-1})}{\alpha} - \frac{\gamma}{\alpha} - \frac{u_{t-1}}{\alpha}\} \\ &= \lambda \Delta p_t + \frac{(1-\lambda)}{\alpha}(m_{t-1} - p_{t-1}) - \frac{(1-\lambda)\gamma}{\alpha}\alpha - \frac{(1-\lambda)}{\alpha}u_{t-1} \end{aligned}$$

Plug in (5), we obtain:

$$\begin{aligned} m_t - p_t &= \gamma + \alpha\lambda \Delta p_t + (1-\lambda)(m_{t-1} - p_{t-1}) - (1-\lambda)\gamma - (1-\lambda)u_{t-1} + u_t \\ &= \lambda\alpha + \alpha\lambda \Delta p_t + (1-\lambda)(m_{t-1} - p_{t-1}) - (1-\lambda)u_{t-1} + u_t \end{aligned} \tag{6}$$

8- • We observe α negative and λ lies between 0 and 1.
• The fit is good (R^2 are high).
• The λ's are small: the expectations are very sticky, persistent.

9- (6)$\Leftrightarrow$

$$m_t - p_t = \lambda\alpha + \alpha\lambda p_t - \alpha\lambda p_{t-1} + (1-\lambda)m_{t-1} - (1-\lambda)p_{t-1} - (1-\lambda)u_{t-1} + u_t$$

$$p_t(1+\alpha\lambda) = (1+\alpha\lambda-\lambda)p_{t-1} + m_t - (1-\lambda)m_{t-1} + (1-\lambda)u_{t-1} - \lambda\gamma$$

If $m_t = m$, $u_t = 0$ for all t

$$p_t = \frac{1+\alpha\lambda-\lambda}{1+\alpha\lambda}p_{t-1} + \lambda m - \lambda\gamma$$

Stable if $|\frac{1+\alpha\lambda-\lambda}{1+\alpha\lambda}| < 1$

10- In Germany and Russia, one can have hyper inflation with constant money growth. If this model is correct, hyperinflation is not always a monetary phenomenon, caused by too expansionist monetary policy. It can be driven by expectations.

11- The problem with adaptive expectations is that agents make forecastable prediction errors.

12- $\pi_t = \Delta p_{t+1}$

Now the model solution is the solution of:

$$m_t - p_t = \gamma + \alpha(p_{t-1} - p_t) + u_t \tag{7}$$

with $m_t = m,\ u_t = 0\ \forall t$.

$$\begin{aligned} \alpha p_{t+1} &= (\alpha-1)p_t + m - \gamma \\ p_{t+1} &= \tfrac{\alpha-1}{\alpha}p_t + \tfrac{m-\gamma}{\alpha} \end{aligned} \tag{8}$$

where $\frac{1-\alpha}{\alpha} = 1 - \frac{1}{\alpha} > 1$ if $\alpha < 0$.

Let's assume $\alpha < 0$ for now on.

13- Long run value $\overline{p}$

$$\begin{aligned} \overline{p} &= (1-\tfrac{1}{\alpha})\overline{p} + m - \gamma \\ \tfrac{1}{\alpha}\overline{p} &= m - \gamma \\ \Rightarrow \overline{p} &= \alpha(m-\gamma) \end{aligned} \tag{9}$$

And:

$$\begin{aligned} p_{t+1} &= \tfrac{\alpha-1}{\alpha}p_t + m - \gamma \\ \Leftrightarrow (p_{t+1} - \overline{p}) &= \tfrac{\alpha-1}{\alpha}(p_t - \overline{p}) + \tfrac{1}{\alpha}\overline{p} + m - \gamma \\ \Leftrightarrow (p_{t+1} - \overline{p}) &= \tfrac{\alpha-1}{\alpha}(p_t - \overline{p}) \end{aligned} \tag{10}$$

14- Therefore,

$$\begin{aligned} (p_{t+1} - \overline{p}) &= (\tfrac{\alpha-1}{\alpha})^{t+1}(p_0 - \overline{p}) \\ p_{t+1} &= (\tfrac{\alpha-1}{\alpha})^{t+1}(p_0 - \overline{p}) + \overline{p} \end{aligned} \tag{11}$$

15- if $p_0 \neq \overline{p}$, $\lim_{t\to\infty} p_{t+1} = \pm\infty$ or if $p_0 = \overline{p}$, $p_{t+1} = \overline{p}\ \forall t$. $\rightsquigarrow$ see figure 6

16- Let $\widehat{p} = \alpha(\widehat{m} - \gamma)$ and $\widetilde{p} = \alpha(\widetilde{m} - \gamma)$ $\rightsquigarrow$ see figure 7

17-

$$\pi_t = E_t[p_{t+1}] - p_t \tag{12}$$

Then

$$\begin{aligned} m_t - p_t &= \gamma + \alpha(E_t[p_{t+1}] - p_t) + u_t \\ (1-\alpha)p_t &= -\alpha E_t[p_{t+1}] - \gamma + m_t - u_t \end{aligned} \tag{13}$$

18-

$$\Leftrightarrow (1-\alpha)p_{t+1} = -\alpha E_{t+1}p_{t+2} - \gamma + m_{t+1} - u_{t+1} \tag{14}$$

Take E_t implies:

$$\begin{aligned} (1-\alpha)E_t p_{t+1} &= -\alpha E_t p_{t+2} - \gamma + E_t m_{t+1} \\ \Leftrightarrow E_t(p_{t+1}) &= -\tfrac{\alpha}{1-\alpha}E_t[p_{t+2}] - \tfrac{\gamma}{1-\alpha} + \tfrac{E_t[m_{t+1}]}{1-\alpha} \end{aligned} \tag{15}$$

Figure 6: The Equilibrium Price Jumps on its Stationary Value

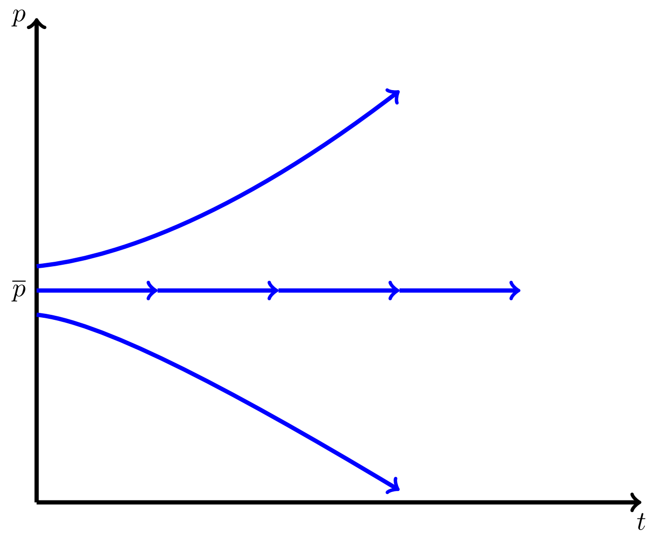
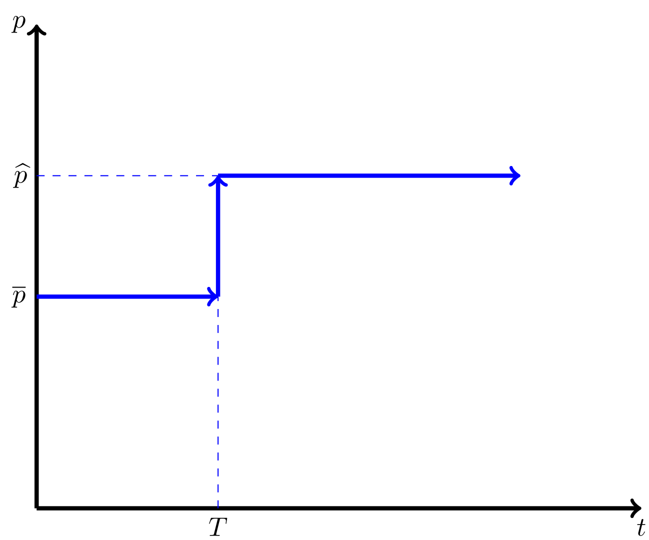


Figure 7: The Effect of a Permanent Shock



19- (13)$\Leftrightarrow$

$$p_t = -\frac{\alpha}{1-\alpha}E_t[p_{t+1}] - \frac{\gamma}{1-\alpha} + \frac{m_t}{1-\alpha} - \frac{u_t}{1-\alpha} \tag{16}$$

using (15) and solving forward:

$$\begin{aligned} p_t &= -\tfrac{\alpha}{1-\alpha}\{-\tfrac{\alpha}{1-\alpha}E_t[p_{t+2}] - \tfrac{\gamma}{1-\alpha} + \tfrac{E_t[m_{t+1}]}{1-\alpha}\} - \tfrac{\gamma}{1-\alpha} + \tfrac{m_t}{1-\alpha} - \tfrac{u_t}{1-\alpha} \\ &= ... \\ &= \tfrac{1}{1-\alpha}\{m_t - \gamma\{1 - \tfrac{\alpha}{1-\alpha} + (\tfrac{\alpha}{1-\alpha})^2 + ...\} - u_t + \tfrac{\alpha}{1-\alpha}E_t m_{t+1} + (\tfrac{\alpha}{1-\alpha})^2 E_t m_{t+2} + ...\} \end{aligned} \tag{17}$$

20- The current price level is a function of all expected future.

21- persistence in money supply.

22-

$$\begin{aligned} m_t &= \mu_0 + \mu_1 m_{t-1} + e_t & \\ E_t(m_{t+1}) &= E_t(\mu_0 + \mu_1 m_t + e_{t+1}) &= \mu_0 + \mu_1 m_t \\ E_t(m_{t+2}) &= E_t(\mu_0 + \mu_1 m_{t+1} + e_{t+2}) &= \mu_0 + \mu_1\mu_0 + \mu_1^2 m_t \end{aligned} \tag{18}$$

23- Money demand shock: u_t. $\frac{\Delta p_t}{\Delta u_t} = -\frac{1}{1-\alpha} < 0$. A positive shock on real money demand for a r and m_t given, $\Rightarrow$ we need real money supply to increase to reach equilibrium $\Rightarrow$ P_t decreases.

24- $\frac{\partial p_t}{\partial m_t} = \frac{1}{1-\alpha-\mu_1\alpha} = \frac{1}{1-(1-\mu_1)\alpha} > 0$ if $\mu_1 = 1$, $\frac{\partial p_t}{\partial m_t} = 1$

25-Quantitative theory of money $py = mv$, if $y = v = cste$, $\frac{\partial p}{\partial m} = 1$. If $\mu_1 < 1$ $\frac{\partial p}{\partial m} < 1$ because the money demand is affected (if it corresponds to a change in v).

Problem IV – Identification in VARs

1 – Consider VAR with includes output, prices, nominal interest rates and money, $Y_t = [GDP_t, P_t, i_t, M_t]$. There are four shocks in the economy. Suppose that a class of models suggests that output contemporaneously reacts only to its own shocks; that prices respond contemporaneously to output and money shocks; that interest rates respond contemporaneously only to money shocks, while money contemporaneously responds to all shocks. Are the four structural shocks identifiable?

Solution : Using the notations of the course, $\nu = A(0)\varepsilon$, where ν is the vector of structural shocks, ε the vector of non structural (and non orthogonal) shocks. $A(0)$ represents the impact effect of shocks on the macro variables. Let's denote a_{ij} the line i column j element of A(0). For example, a_{12} is the impact effect on output of the money shock. Knowing Ω the variance-covariance matrix of ε, we have

$$V(A(0)\varepsilon) = V(\nu) \Longleftrightarrow A(0)A(0)' = \Omega$$

This give us 10 independent equations and we need to find the 16 coefficients of $A(0)$. The restrictions stated in the question imply some zeros in the $A(0)$ matrix:

$$A(0) = \begin{pmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$$

Note that we are left with 10 coefficients, which is the number of equations that we have. The structural shocks can then be identified.

2 – Suppose we have extraneous information which allows us to pin down some of the parameters of the matrix $A(0)$ (this notation is the one of chapter 1). For example, suppose in a trivariate system with output, hours and taxes, we can obtain estimates of the elasticity of hours with respect to taxes. How many restrictions do you need to identify the shocks? Does it make a difference if zero or constant restriction is used?

Solution : In this case, the matrix $A(0)$ is 3×3 so that 9 coefficients are to be found. 6 equations are given by the equality of variance of ν and $A(0)\varepsilon$. We then need 3 more restrictions. Assume that we want to identify an output shock, an hour shock and a tax shock (in that order). Knowing the elasticity of hours to a tax shock (say it is k) implies that $a_{23} = k$, which already give one restriction. Two other will needed to identify the 3 shocks. Note that for identification, it does not matter whether the restriction is a zero or a real non zero number (i.e. whether k is null or not).