Toulouse School of Economics, 2009-2010
Macroeconomics II – Franck Portier

Homework 1

Problem I – An AD-AS Model

Let us consider an economy with three agents (a firm, a household and a government) and four goods (consumption and investment good Y, labor L, money M and bonds B). We assume that the firm self-finance its investment, so that the only bonds are public bonds. All constants are positive. It is assumed that expected inflation is null.

The production function is given by

$$Y = AL^\alpha$$

and we asssume that the firm maximizes its nominal profit $PY - WL$, where W us the nomianl wage and P the general price level. Investment is given by

$$I = \overline{I} - \gamma i$$

where i is the nominal interest rate.

The household's consumption is

$$C = c\mathcal{Y} + \overline{C},$$

where disposible income $\mathcal{Y}$ is

$$\mathcal{Y} = (1-\tau)Y - T$$

(T is a lump sum transfert). We assume that labor supply is inelastic at level $L^s = \overline{L}$, and that household's money demand is :

$$M^d = P(\alpha Y - \beta i)$$

The government demands G on the good market and receives tax revenues $\tau Y + T$. It supplies an amount of money $M^s = \overline{M}$, and issues a level B of debt.

⋆

We first construct the Aggregate Supply curve, assuming that prices and wages are fully flexible.

1 – Compute labour demand.

2 – Draw labor demand and labor supply. Show the determination of the equilibrium real wage. Compute the equilibrium levels of real wage, labor, output.

3 – Draw the AS curve Y^S in the (Y, P) space.

⋆

Let us now construct the Aggregate Demand curve as the solution of the IS-LM model for a given level of prices P.

4 – Define and give the equation of the IS curve.

5 – Define and give the equation of the LM curve.

6 – Compute the equilibrium income of the IS-LM model, which is the AD curve Y^D.

⋆

Let us compute now the equilibrium of the whole model.

7 – Write down the budget constraint of the government. Which policies are available? Are all policy instruments independent?

8 – Draw the AD and AS curves. Compute the equilibrium level of output Y and the value of the government expenditures multiplier $\frac{dY}{dG}$ and of the monetary policy one.

⋆

Assume now that the nominal wage is rigid at level $\overline{W}$.

9 – Find the price $\overline{P}$ for which the labor market clears..

10 – Compute the labor demand as a function of P and consider the two regimes $P < \overline{P}$ and $P > \overline{P}$.

11 – Draw labor demand and supply in the space (L, P).

12 – Give the expressions of AD and AS and draw them.

13 – Discuss of the multipliers values along the AD locus (ie in IS-LM) and at the AD-AS equilibrium.

Problem II – A Lucas 73 type model

The model: The economy is composed of a large number N of consumers-workers-producers indexed by i. Each good i is produced and sold by a representative agent (consumer-worker-producer) acting in a competitive way. Technology is given by $Q_i = L_i$, where L_i is labor and Q_i good. Agent i decides of consumption and labor to maximize

$$U_i = C_i - \frac{1}{\gamma} L_i^{\gamma}, \quad \gamma > 1$$

subject to the budget constraint $PC_i = P_i Q_i$, where C_i is the basket of goods consumed by agent i and P is the price of this basket.
1 – Find the optimal Q_i and L_i for given prices. Write those equations in logs, with the notations q_i, p, ℓ_i et p_i for the logs of Q_i, P, L_i et P_i. The equation of q_i will be referred to as supply of good i.

Let us assume that the demand of good i takes the following form:

$$q_i = y + z_i - \eta(p_i - p) \tag{1}$$

Demand of good i is a function of three terms: 1) average real income y, 2) sector i specific shock z_i, 3) relative price of good i. We assume that the z_i are iid across sectors, normally distributed with mean 0 and variance V_z. By definition, $y = \frac{1}{N}\sum_i q_i$ and $p = \frac{1}{N}\sum_i p_i$. We assume that N is large, so that $\frac{1}{N}\sum_i z_i = 0$.
Aggregate demand is given by

$$y = m - p \tag{2}$$

where m is money supply per capita. We assume that m is normally distributed with mean $E[m]$ and variance V_m.

Perfect Information : Let us assume first that z_i et m are observable by all agents.
2 – Compute equilibrium prices and quantities. What is the value of $\frac{\partial y}{\partial m}$? Comment.

Imperfect Information : Let us assume now that agent i observes p_i but not p. This agent will form a rational expectation of the relative price $r_i = p_i - p$ conditionally to the observation of p_i and the knowledge of the model. This expectation is denoted $E[r_i|p_i]$. We assume that the production decision is taken according to this expectation, so that

$$q_i = \ell_i = \frac{1}{\gamma - 1} E[r_i|p_i] \tag{3}$$

3 – We assume that r_i and p are normally distributed (we'll check that later). Under this assumption, one can show that the signal extraction formula is given by

$$E[r_i|p_i] = \frac{V_r}{V_r + V_p}(p_i - E[p])$$

Comment this expression. What does happen in the limit case $V_p = 0$.
4 – Compute the aggregate supply curve (i.e. the expression of y) and comment this "Lucas supply curve".

With imperfect information, the model is given by

$$\begin{aligned} q_i &= b(p_i - E[p]) && (4)\\ y &= b(p - E[p]) && (5)\\ y &= m - p && (6) \end{aligned}$$

with $b = \frac{1}{\gamma-1}\frac{V_r}{V_r+V_p}$.
5 – Compute the rational expectation of the equilibrium price $E[p]$. Then compute the equilibrium values of p and y for a given b.
6 – What is the value of $\frac{\partial y}{\partial m}$? Is the distinction between anticipated and non anticipated monetary policy important?
7 – Compute the variance of p, V_p.

8 – Compute $r_i = p_i - p$ using (4) and (5). What is the value of V_r?
9 – From the two last equations, derive an implicit equation in b. Compute explicitly b when $\eta = 1$.
10 – Check that p and r_i are normally distributed, as supposed in question 3.

We have just shown that the solution of the model was

$$p = E[m] + \frac{1}{1+b}(m - E[m]) \tag{7}$$

$$y = \frac{b}{1+b}(m - E[m]) \tag{8}$$

Let us assume that m follows a random walk with drift:

$$m_t = m_{t-1} + c + u_t$$

where u_t is iid with zero mean and c a positive constant.

11 – Derive the equilibrium process of p_t and y_t? What is the process of inflation $\pi_t = p_t - p_{t-1}$?

12 – What can be said about the slope of the Phillips Curve in this model (i.e. the correlation between inflation and output). What is the consequence on output of a permanent increase in c? Comment.

Problem III – Cagan's model

In 1956, P. Cagan published a study of hyper inflation episodes, episodes in which expectations seem to be crucial. This problem is inspired from Cagan's model, but modifies the way expectations are modelled. We assume that real variables (output, interest rate) are constant, and we study the interactions between the general price level and the money supply, using a money demand equation.

⋆

1 – The Cagan's money demand equation is specified as follows::

$$\log(M_t/P_t) = \alpha_0 + \alpha_1 \log y_t + \alpha_2 R_t + u_t$$

where M is the monetary aggregate, P the price level, R the nominal interest rate and u_t an *iid* random shock with zero mean. Comment this equation. What are a priori the signs of the coefficients?

2 – Given the relation $R_t = r_t + \pi_t$, where r is the real interest rate and π the expected inflation, and under the assumption (admissible in the very short run) $y_t = y, r_t = r \;\; \forall t$, show that the money deman can be written

$$m_t - p_t = \gamma + \alpha\pi_t + u_t \tag{9}$$

where m et p are the natural logarithms of M and P, and γ, α some constants to define.

Table 1: Cagan's data (1956)

Country	Period	Average inflation rate (% per month)	Real balances (minimum/initial)
Austria	Oct 1921–Aug 1922	47.1	.35
Germany	Aug 1922–Nov 1923	322	.030
Greece	Nov 1943–Nov 1944	365	.007
Hungary	March 1923–Feb 1924	46.0	.39
Hungary	Aug 1945–June 1946	19800	.0003
Poland	Jan 1923–Jan 1924	81.1	.34
Russia	Dec 1921–Jan 1924	57.0	.27

3 – In table 1 are gathered Cagan's observations concerning hyper inflation episodes. The last column presents the minimum level of real balances M/P over the period, as a % of the initial level of real balances. Comment this table. Are these observations compatible with the money demand equation?

⋆

4 – Why can't we estimate directly equation 9 using available data?

5 – Working under the supervision of Friedman, Cagan assumed that expectations were adaptative:

$$\pi_t - \pi_{t-1} = \lambda \left(\Delta p_t - \pi_{t-1}\right)$$

with $\Delta p_t = p_t - p_{t-1}$ and $\pi_t = p^a_{t+1} - p_t$, where p^a_{t+1} is the expectation of the price level in $t+1$. Comment this expectation equation.

6 – Show that expected inflation π_t can be written as a weighted sum of past inflation rates Δp_{t-i}. What is the meaning of the λ parameter?

7 – Using the expectation formula given in 5), solve the model to get $m_t - p_t$ as a fi=unction of observable variables Δp_t, $m_{t-1} - p_{t-1}$, u_t and u_{t-1}.

Table 2: Cagan's estimates

Episode	α	λ	R^2
Austria	-8.55	.05	.978
Germany	-5.46	.20	.984
Greece	-4.09	.015	.960
Hungary	-8.7	.10	.857
Hungary	-3.63	.15	.996
Poland	-2.3	.3	.945
Russia	-3.06	.35	.942

8 – Table 2 shows Cagan's results from estimating α and λ. Comment.

★

9 – Assume that money supply is exogenous. Rewrite the model's solution as $p_t = f\left(p_{t-1}, m_t, m_{t-1}, u_t, u_{t-1}\right)$. We further assume that $m_t = m$ et $u_t = 0 \;\; \forall t$. Under which condition is the model stable (ie converging to a finite limit)?

10 – Cagan's estimations lead to the results given in table 3. Are there countries in which hypo er inflation can occur without any explosion of the money supply. why?

Table 3: Cagan's estimates

Episode	$\alpha\lambda + 1 - \lambda$	$1 + \alpha\lambda$	$\frac{\alpha\lambda+1-\lambda}{1+\alpha\lambda}$
Austria	.516	.556	.928
Germany	-.292	-.092	3.17
Greece	.236	.386	.611
Hungary	.03	.13	.23
Hungary	.305	.455	.67
Poland	.01	.31	.032
Russia	-.421	-.07	5.92

★

11 – Why is adaptative expectations not such a good assumption?

12 – Assume now that there is no uncertainty and that expectations are perfect ($\pi_t = \Delta p_{t+1}$) and that $m_t = m$ et $u_t = 0 \;\; \forall t$). Write down the model's solution as a difference equation in p_{t+1} and p_t.

13 – Compute the long run value of the equilibrium price $\overline{p}$, and rewrite the equation with the variables $p_t - \overline{p}$ and $p_{t+1} - \overline{p}$.

14 – What is the equation's solution for an arbitrary initial condition p_0?

15 – What is the value of p_0 that ensures convergence towards $\overline{p}$? Comment.

16 – For the rest of the problem, we assume $\alpha < 0$. We also assume that $m_t = \widehat{m}\ \ \forall t \in [0, T]$ et $m_t = \widetilde{m}\ \ \forall t \in [T, +\infty[$, with $\widetilde{m} > \widehat{m}$. Draw on a graph the path of p_t. Comment.

★

17 – We assume now that there is uncertainty and that expectations are rational: $\pi_t = E_t\left[p_{t+1}\right] - p_t$. Unsing money demand equation, compute p_t as a function of m_t, u_t and $E_t\left[p_{t+1}\right]$.

18 – To solve forward this equation, find a relation between $E_t\left[p_{t+1}\right]$, $E_t\left[p_{t+2}\right]$ and $E_t\left[m_{t+1}\right]$.

19 – How can we then get the following equation (explain but don't report the computation)?

$$p_t = \frac{1}{1-\alpha}\left(m_t - \gamma(1-\alpha) - u_t + \frac{\alpha}{\alpha - 1}E_t\left[m_{t+1}\right] + \left(\frac{\alpha}{\alpha-1}\right)^2 E_t\left[m_{t+2}\right] + \cdots\right) \tag{10}$$

20 – Comment this equation.

21 – We assume now that money supply is given by:

$$m_t = \mu_0 + \mu_1 m_{t-1} + e_t$$

with $|\mu_1| \leq 1$, $\mu_0 > 0$ and where e is an *iid* shock with zero mean. Comment this money supply.

22 – Compute $E_t\left[m_{t+1}\right]$, $E_t\left[m_{t+2}\right]$,...,$E_t\left[m_{t+j}\right]$.

23 – After some tedious calculus (that you could try to do by yourself), on,e gets the following solution of the model:

$$p_t = \frac{-\alpha\mu_0}{1-\alpha+\alpha\mu_1} - \gamma + \frac{1}{1-\alpha+\alpha\mu_1}m_t - \frac{1}{1-\alpha}u_t \tag{11}$$

or equivalently

$$p_t = \frac{-\alpha\mu_0}{1-\alpha+\alpha\mu_1} - \gamma + \frac{\frac{\mu_0}{1-\mu_1} + e_t + \mu_1 e_{t-1} + \cdots}{1-\alpha+\alpha\mu_1} - \frac{1}{1-\alpha}u_t \tag{12}$$

What is the consequence on prices of a positive shock u_t? What does mean this shock?

24 – What is the effect on prices of a monetary shock m_t? What does happen when $\mu_1 = 1$, when $0 < \mu_1 < 1$?

25 – Can we say from those results that quantitative theory of money (look in Romer if you do not know what it means) does not apply when the shock is not permanent?

Problem IV – Identification in VARs

1 – Consider VAR with includes output, prices, nominal interest rates and money, $Y_t = [GDP_t, P_t, i_t, M_t]$. There are four shocks in the economy. Suppose that a class of models suggests that output contemporaneously reacts only to its own shocks; that prices respond contemporaneously to output and money shocks; that interest rates respond contemporaneously only to money shocks, while money contemporaneously responds to all shocks. Are the four structural shocksw identifiable?

2 – Suppose we have extraneous information which allows us to pin down some of the parameters of the matrix $A(0)$ (this notation is the one of chapter 1). For example, suppose in a trivariate system with output, hours and taxes, we can obtain estimates of the elasticity of hours with respect to taxes. How many restrictions do you need to identify the shocks? Does it make a difference if zero or constant restriction is used?