CHAPTER 3
REAL BUSINESS CYCLES

- Main Reference:

  – R. King and S. Rebelo, "Resuscitating Real Business Cycles",

Handbook of Macroeconomics, 2000,

- Other references that could be read :

  – Blanchard and Fisher [1989], Chapter 7,

  – Romer [2001], Chapter 4

1   Introduction

The modern approach to fluctuations is presented here

• I present here the simplest version of a model that has been extensively used to model Business Cycle over the past 30 years, since Kydland & Prescott (1982).

• The main features of this model are : intertemporal general equilibrium, stochastic model, role of technological shocks

• All along, the name of the game is to reproduce some "stylized facts" of the business cycle.

• The model should be seen as illustrating a powerful tool: DSGE (Dynamic Stochastic General Equilibirum model)

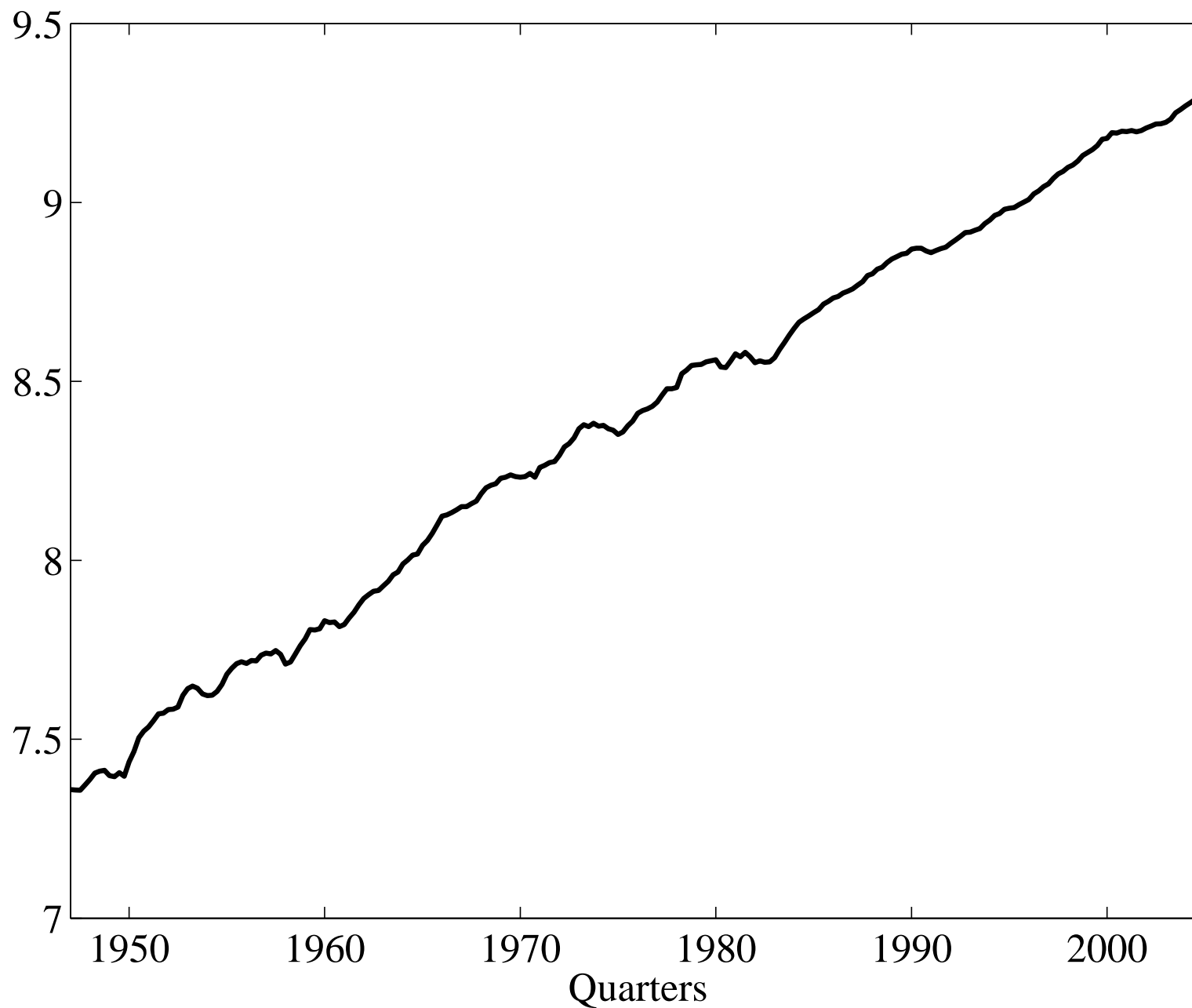# 2 Measuring the Business Cycle

2.1 Trend versus Cycle

- Any Time Series can be decomposed as

$$x_t = x_t^T + x_t^c$$

- Problem: How is define/identify each component?

Figure 1: US log GDP per capita

- Several ways of approaching the problem

- Actually: Infinite number of decomposition of a non-stationary process into a cycle and a trend

- Let us see some of those decompositions

2.1.1 Cycle: Output Gap

- Defined as

$$\text{Actual output} - \text{Potential Output}$$

- Expansion: Actual output $\geqslant$ Potential output

- Actual output: easy to observe

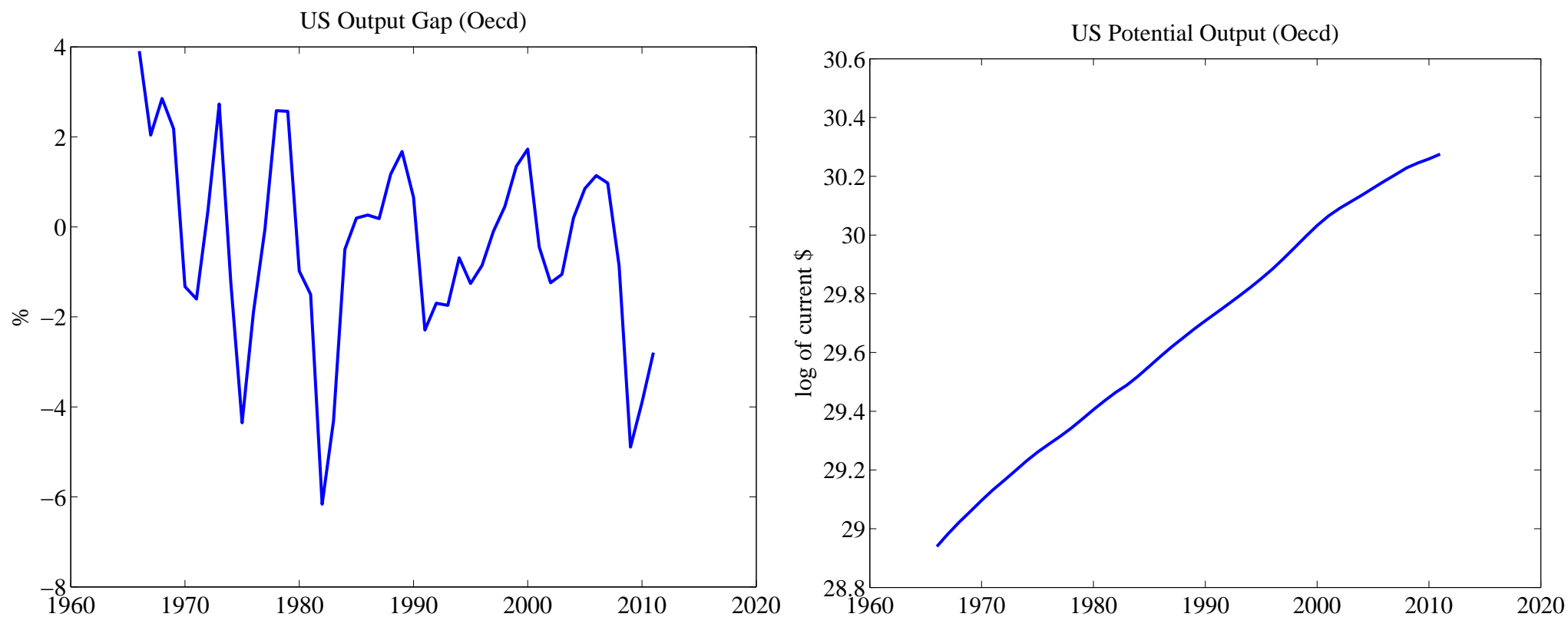- Note: How to identify potential output? (full utilization?, efficient?)

- Example:

  (1) estimate $y_t = \alpha \ \times \ u_t + \text{other controls} + \varepsilon_t,$

  (2) define potential output as $y_t^P = \widehat{\alpha}_t \times 0\% + \text{other controls} + \widehat{\varepsilon}_t.$

- This is an over simplified description of the method used by Oecd.
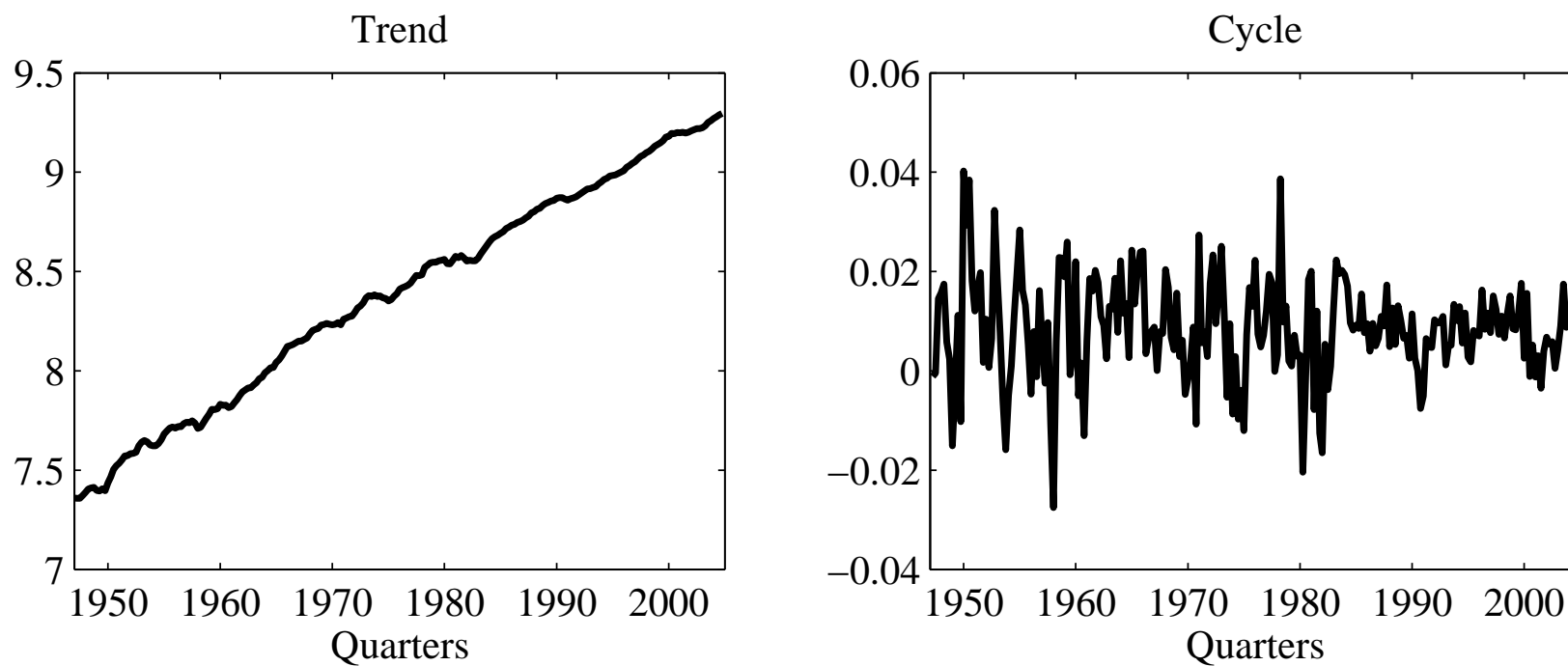
Figure 2: US Output Gap and Potential Output

2.1.2 Growth Cycle

- Take the growth rate of the series

- Expansion: Positive rate of growth

- Note: the cycle is very volatile (almost iid) – a lot of medium run fluctuations are eliminated
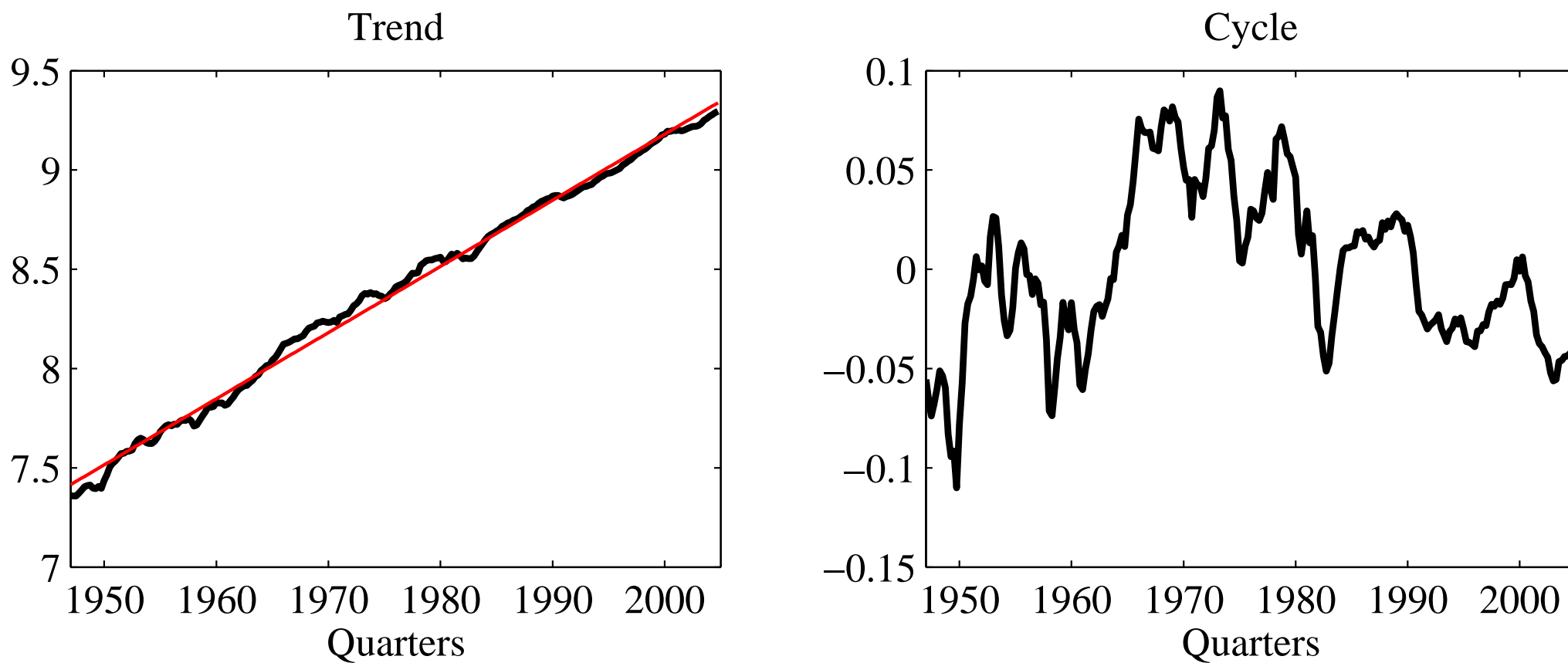
Figure 3: US Growth Cycles

Trend

Cycle

2.1.3 Trend Cycle

- Deviation from linear trend

- The trend is obtained from linear regression

$$\log(x_t) = \alpha + \beta t + u_t$$

- Cycle: $\widehat{x}_t = \log(x_t) - (\widehat{\alpha} + \widehat{\beta}t)$

- Expansion: Output above the trend

- Note: the cycle can be large and very persistent - a lot of medium and long run fluctuations are not eliminated

Figure 4: US Trend Cycles

2.1.4   The Hodrick–Prescott Filter

- Hodrick and Prescott [1980]

- Obtained by solving

$$\min_{\{x_\tau^T\}_{\tau=1}^t} \sum_{\tau=1}^t \left(x_\tau - x_\tau^T\right)^2$$

subject to

$$\sum_{\tau=2}^{t-1} \left(\left(x_{\tau+1}^T - x_\tau^T\right) - \left(x_\tau^T - x_{\tau-1}^T\right)\right)^2 \leqslant c$$

- or

$$\min_{\{x_\tau^T\}_{\tau=1}^t} \sum_{\tau=1}^t \left(x_\tau - x_\tau^T\right)^2 + \lambda \sum_{\tau=2}^{t-1} \left(\left(x_{\tau+1}^T - x_\tau^T\right) - \left(x_\tau^T - x_{\tau-1}^T\right)\right)^2$$
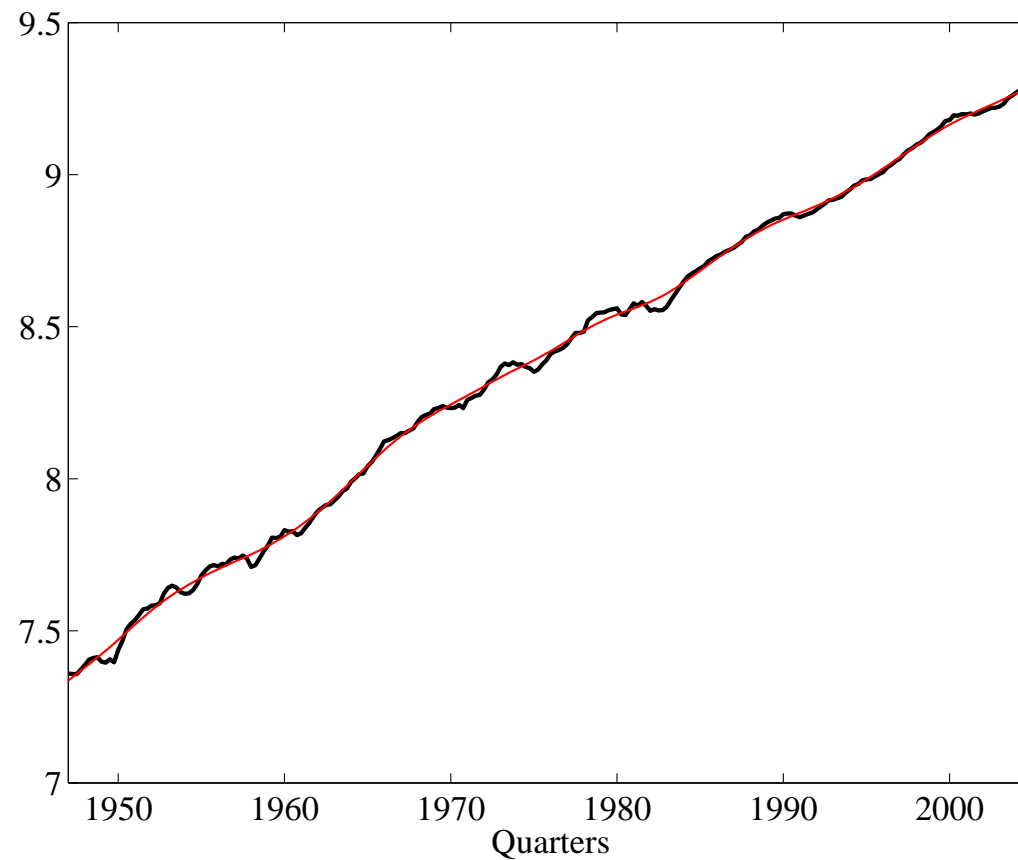
- $\lambda = 0$: the trend is equal to the series.

- $\lambda = \infty$: the trend is linear.

- Setting λ for quarterly data: Accept cyclical variations up to 5% per quarter, and changes in the quarterly rate of growth of 1/8% per quarter, then

$$\lambda = \frac{5^2}{(1/8)^2} = 1600$$

(under some assumptions)
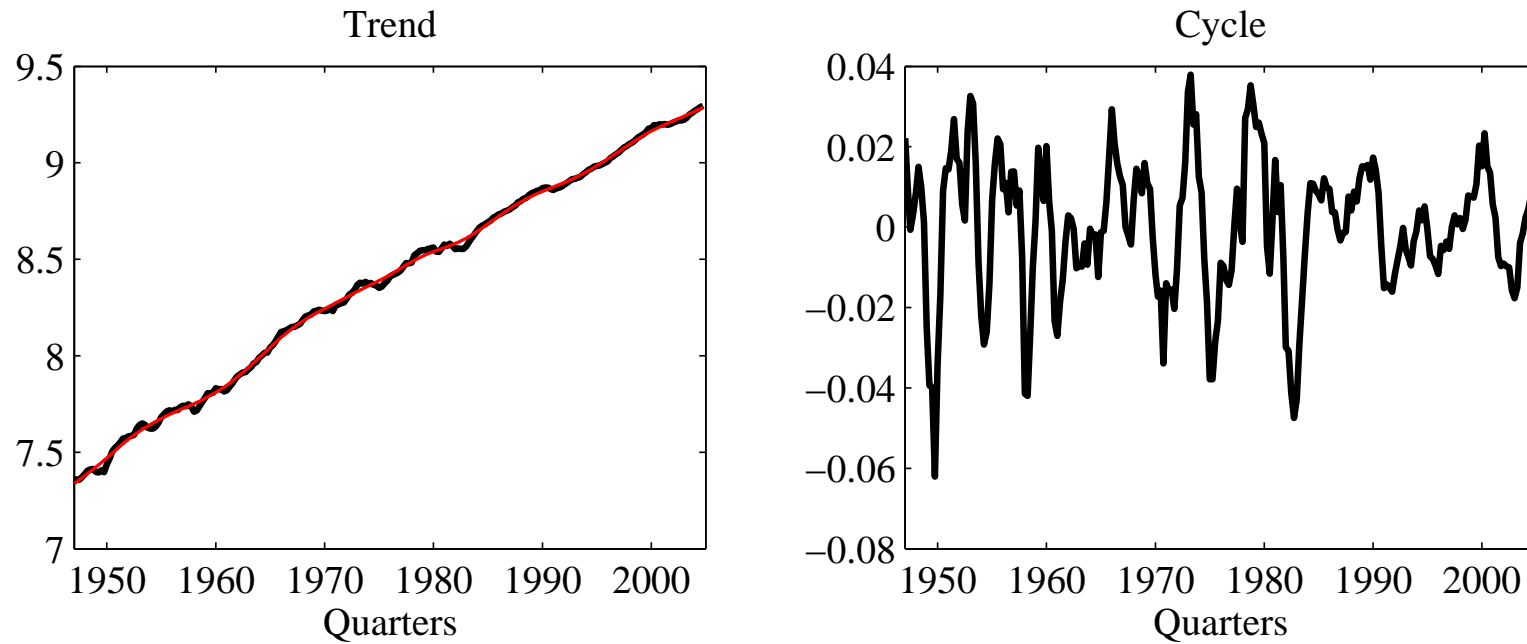
2.1.5   The HP filter at work

Figure 5: US HP Trend

- HP trend: not linear

- cycle is the difference between the two curves

Figure 6: US HP Cycle

- The HP filter is the one mainly used in the literature. We will use it to:

- Get the cyclical component of each macroeconomic time series,

- Compute some statistics to characterize the business cycle.

2.2   U.S. Business Cycles

2.2.1   What are Business Cycles?

- Lucas' definition:

    "Recurrent **fluctuations** of macroeconomic aggregates around trend"
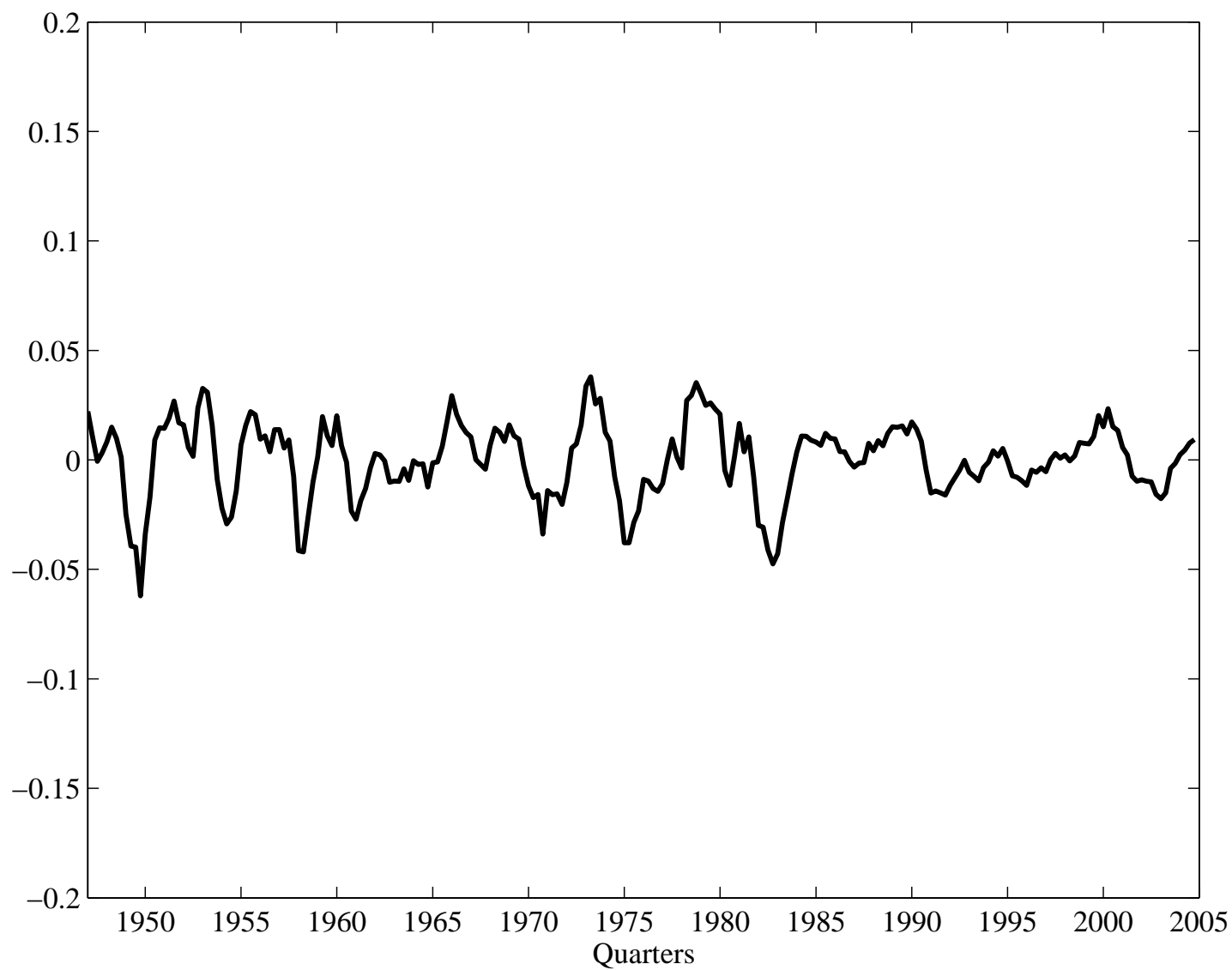
- Want to find **regularities** (*Stylized facts*)

- Business Cycles are characterized by a set of statistics:

    − Volatilities of time series (standard deviations)

– Comovements of time series (correlations, serial correlations)
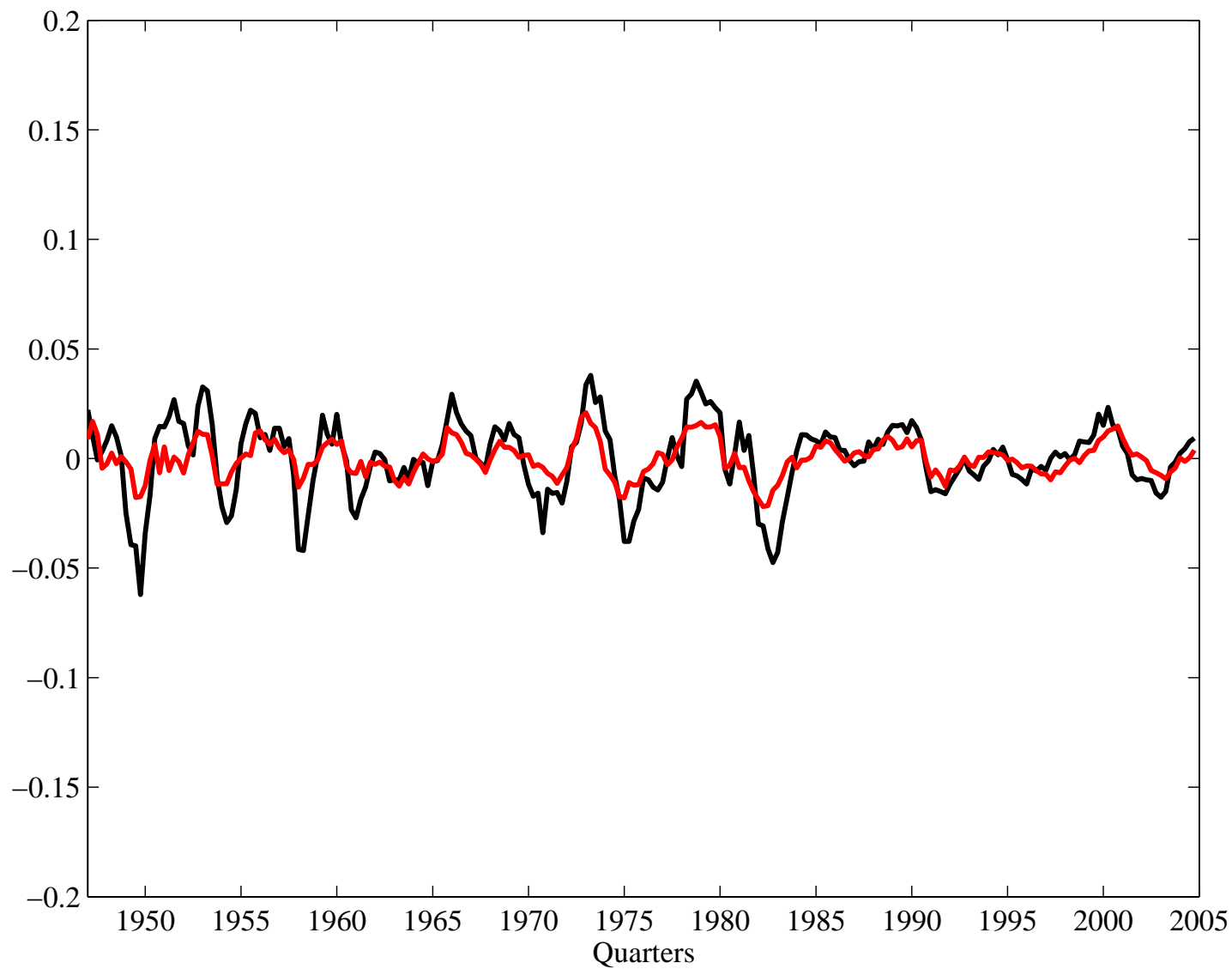
- *"Business Cycles are all alike"*

2.2.2   Main Real Aggregates

- Consumption (C): Nondurables + Services

- Investment (I): Durables + Fixed Investment + Changes in inventories

- Government consumption (G): Absent from the basic model

- Output: $C + I + G$

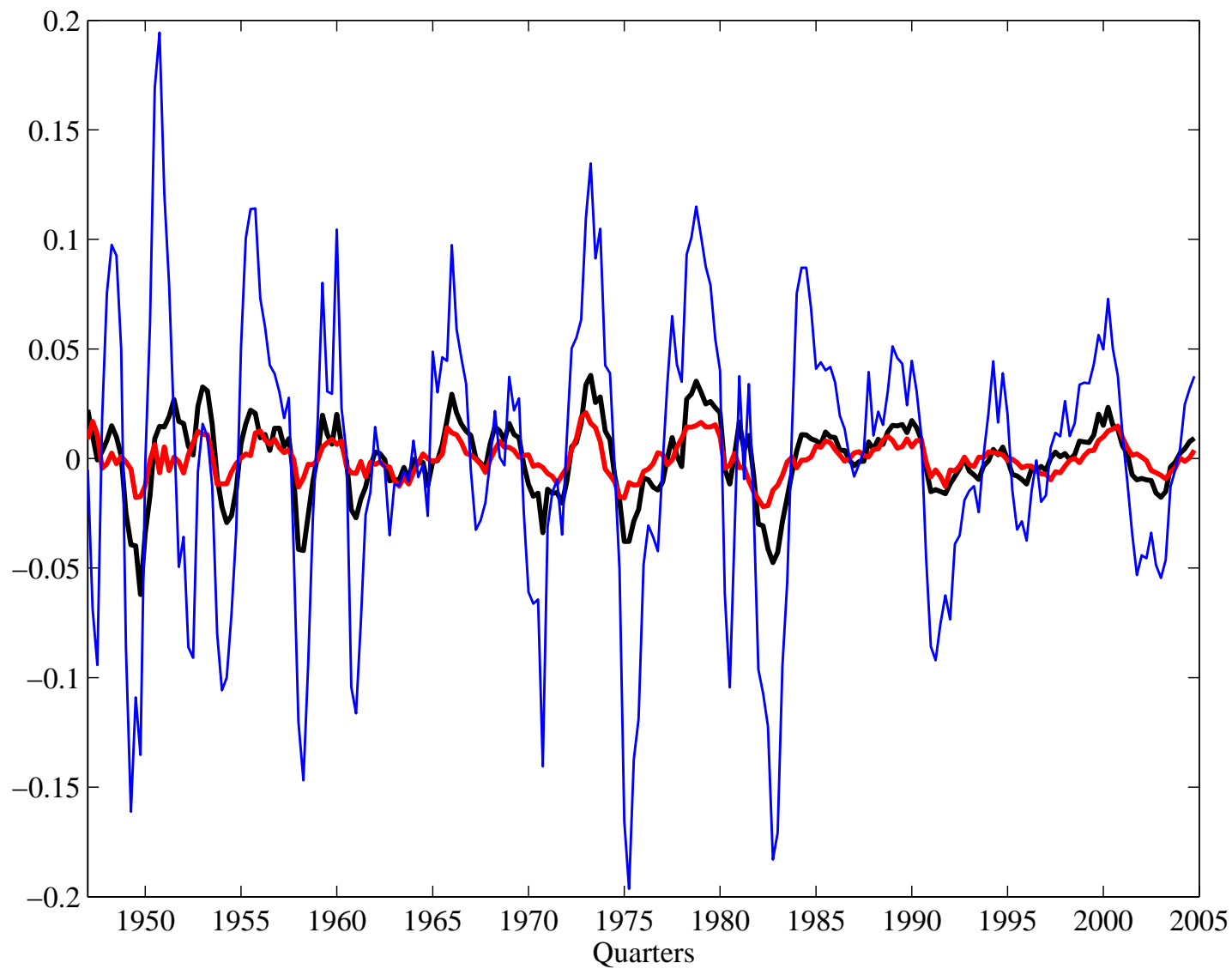- Labor: hours worked

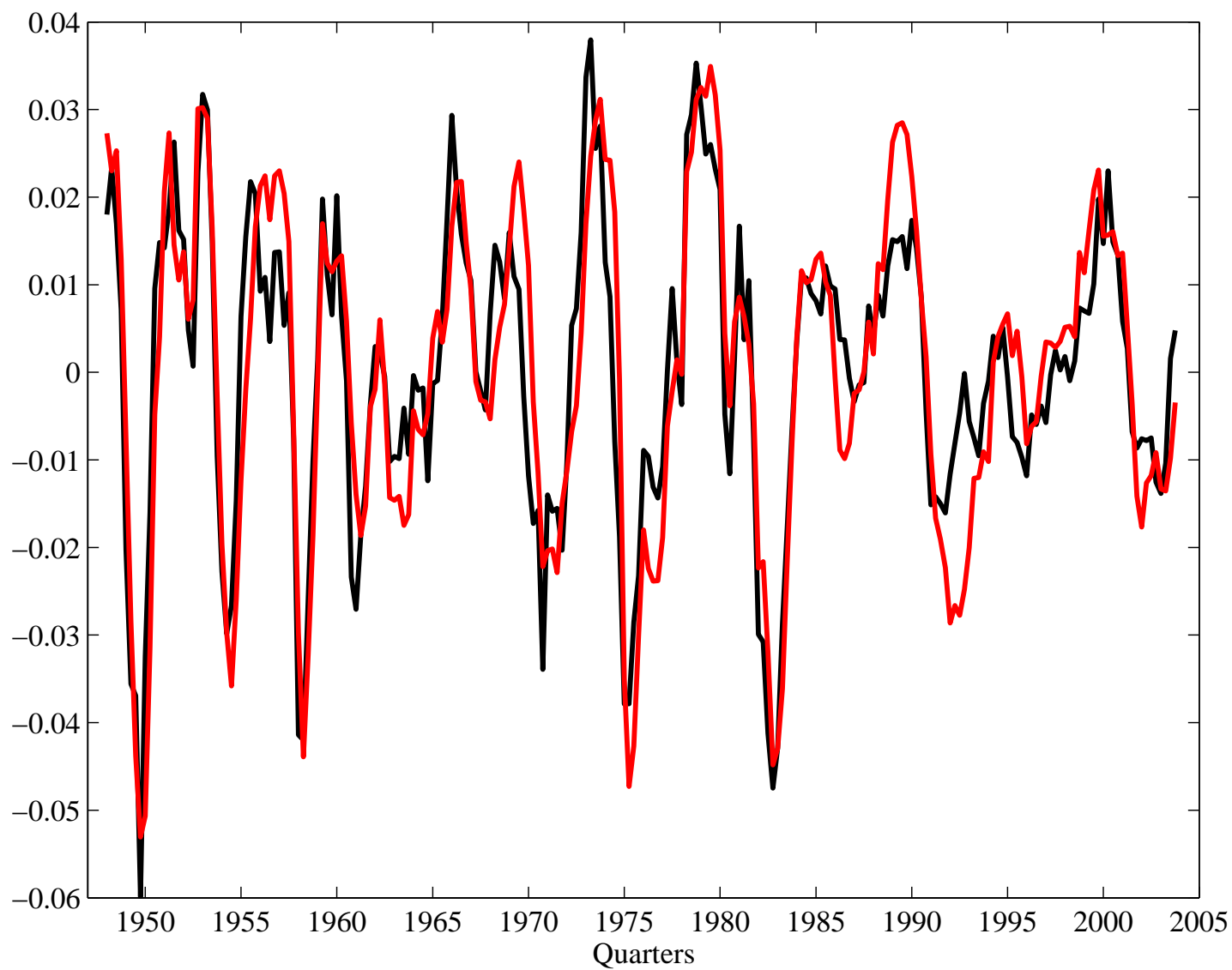- Labor Productivity: Output / Labor
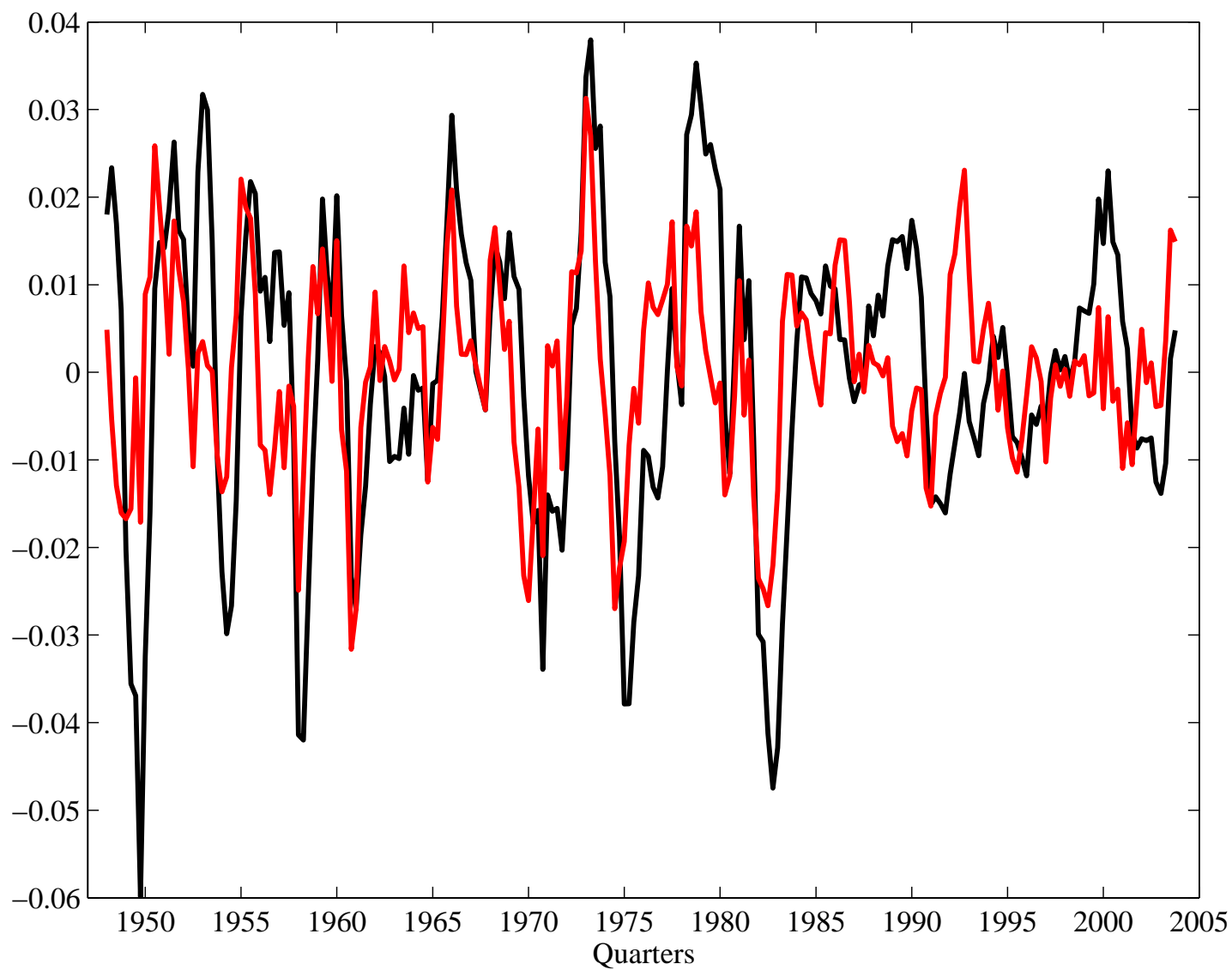
Output – Consumption

Output – Consumption – Investment

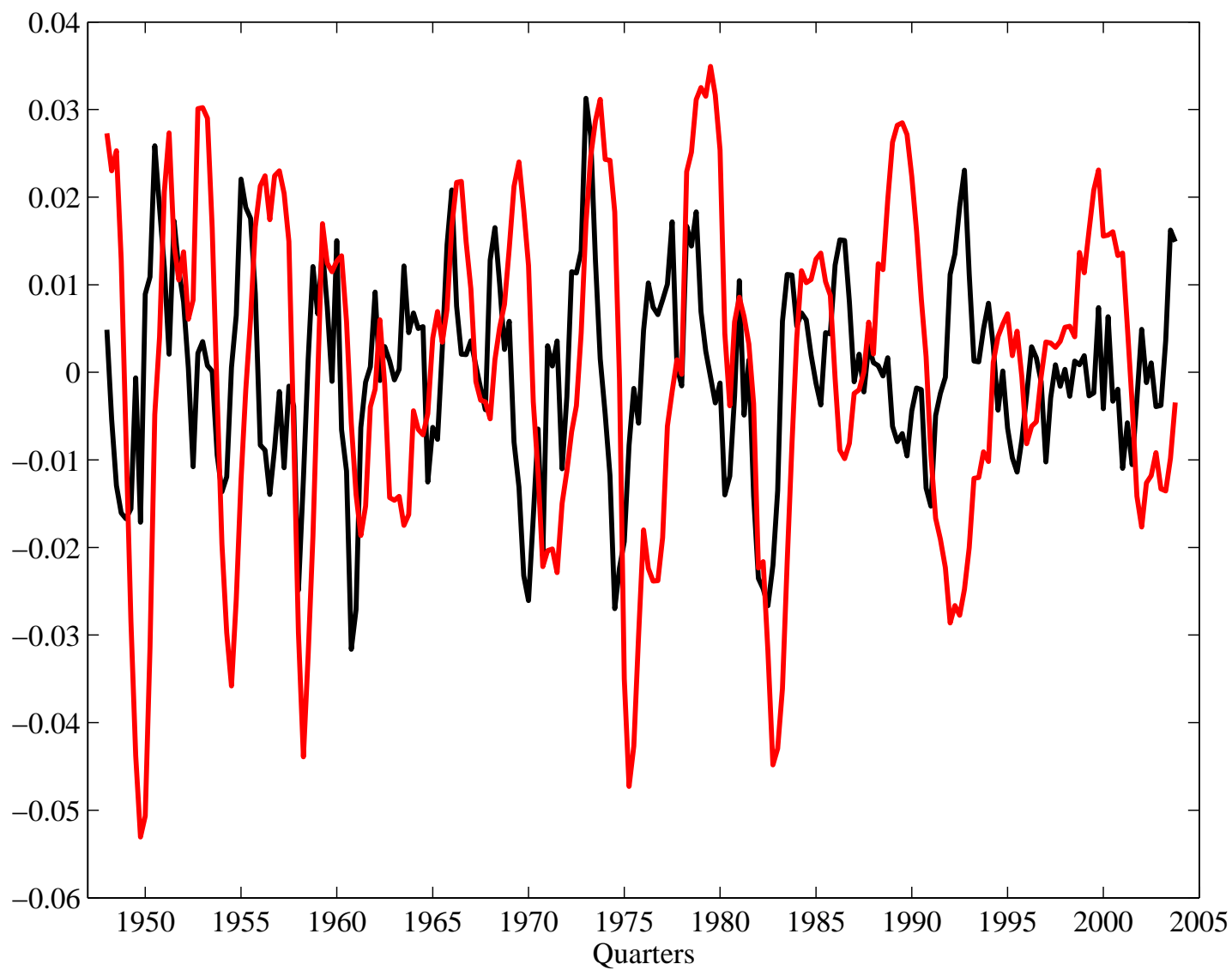# Output − Hours worked

Quarters

Output − Productivity

Productivity – Hours worked

2.2.3 Moments

- We want to characterize fluctuations $\leadsto$ amplitude and movements

- Amplitude: volatilities $\leadsto$ standard deviations

- Comovements: correlations

Variable	$\sigma(\cdot)$	$\sigma(\cdot)/\sigma(y)$	$\rho(\cdot,y)$	$\rho(\cdot,h)$	Auto(1)
Output	1.70	–	–	–	0.84
Consumption	0.80	0.47	0.78	–	0.83
Services	1.11	0.66	0.72	–	0.80
Non Durables	0.72	0.42	0.71	–	0.77
Investment	6.49	3.83	0.84	–	0.81
Fixed investment	5.08	3.00	0.80	–	0.88
Durables	5.23	3.09	0.58	–	0.72
Changes in inventories	22.48	13.26	0.48	–	0.40
Hours worked	1.69	1.00	0.86	–	0.89
Labor productivity	0.90	0.53	0.41	0.09	0.69

Summary

1. Consumption (of non-durables) is less volatile than output

2. Investment is more volatile than output

3. Hours worked are as volatile as output

4. Capital is much less volatile than output

5. Labor productivity is less volatile than output

6. Real wage is much less volatile than output

7. All those variables are persistent and procyclical except Labor productivity that is acyclical

2.3   A Model to Replicate Those Facts

The Facts

Good Market

- Consumption is less volatile than output

- Investment is more volatile than output

- Both are procyclical

- Suggests a Permanent Income component in the model

Labor Market

- Hours is as volatile as output

- Hours are strongly procyclical

- If leisure is countercyclical, why is labor productivity high when labor is high (when we assume decreasing returns to labor)?

- Suggests that productivity shocks might drive the BC

Towards a Model

- Needed: A macro model

- Need labor, consumption, investment ($\leadsto$ Capital)

- Dynamic model

- Can account for growth facts (C, I, Y grow at the same rate in the long run)

- General **equilibrium** model

More specifically:

- Consumption $\rightsquigarrow$ Permanent income component

- Investment $\rightsquigarrow$ Capital accumulation

- Labor $\rightsquigarrow$ Labor market equilibrium

- Shocks to initiate the cycle: Technology shock