Chapter 2

The Limits to the Traditional Approach and the Importance of Dynamics, Expectations, General Equilibrium

- Main references :

  - Ljundqvist and Sargent [2000], Chapter 9 for Ricardian Equivalence

    Benassy, Journal of Economic Literature, [1993], "Nonclearing markets: Microeconomic Concepts and Macroeconomic Applications".

- Other references that could be read :

  - Blanchard and Fisher [1989], Chapter 10,

  - Romer [2001]

1 Introduction

- We explore here some of the reasons why the AD-AS model is not a good tool for policy analysis:

  1. lack of microfoundations in general equilibrium
  2. lack of proper modeling of expectations
  3. lack of dynamics

2 The Importance of the General Equilibrium Consistency

- I have said before that general equilibrium was an important requirement for macroeconomics. Let illustrate this using a very simple general equilibrium model.
- I show with this example (Benassy, JEL, 1993) that interactions between markets are crucial. The example is related to the theory of unemployment.

2.1 The Model

- Consider an economy with two atomistic agents, one representative firm and one representative household.

2.1.1 Preferences and technology

• Firm: $y = \ell^{\beta}$, $0 < \beta < 1$, maximizes profits $\pi = py - w\ell$.

• Household: no disutility of labor supply $\rightsquigarrow$ will inelastically supply ℓ_0,

$$U = \alpha \log(c) + (1 - \alpha) \log(m/p) \quad , 0 < \alpha < 1$$

• The household is endowed with m_0 and receives profits . Its budget constraint is

$$pc + m \leq w\ell + \pi + m_0$$

2.1.2 Markets

- good, labor and money markets
- Both agents behave competitively.

2.1.3 Optimal behaviors

- The Hh maximizes U s.t. the BC:

$$\begin{aligned} \max_{c,m,\ell} \quad & \alpha \log(c) + (1-\alpha)\log(m/p) \\ st \quad & w\ell + \pi + m_0 \geq pc + m \\ & \ell \leq \ell_0 \end{aligned}$$

- Forming the lagrangian,

$$\begin{aligned} \mathcal{L} = {} & \alpha \log(c) + (1-\alpha)\log(m/p) + \lambda(w\ell + \pi + m_0 - pc - m) \\ & + \mu(\ell_0 - \ell) \end{aligned}$$

with $\lambda, \mu \geq 0$ being the Lagrange multipliers.

- From the FOC we get

$$\begin{aligned} c &= \alpha\left(\frac{m_0}{p} + y\right) && (a) \\ m^d &= (1-\alpha)(m_0 + py) && (b) \\ \ell^s &= \ell_0 \end{aligned}$$

- Firm's profit maximization yields

$$l^d = \left(\frac{1}{\beta} \times \frac{w}{p}\right)^{\frac{-1}{1-\beta}}$$

and

$$y^s = \left(\frac{1}{\beta} \times \frac{w}{p}\right)^{\frac{-\beta}{1-\beta}}$$

- Notation: $\frac{w}{p} = \omega$

2.2 Walrasian Equilibrium

• 3 markets: labor, good, money, 2 relative prices (w and p), money being the numéraire.

DEFINITION 1 *A* **Walrasian Equilibrium** *of this economy is a set of prices* (w, p) *and quantities* (c, m, ℓ, y) *such that (i) those quantities maximize utility and profit for those prices and (ii) markets clear.*

- Computing the equilibrium is easy. Labor market equilibrium yields $\ell^\star = \ell_0$, $y^\star = y_0 = \ell_0^\beta$ and $\omega^\star = w^\star/p^\star = \beta\ell_0^{\beta-1}$
- Then the good market equilibrium condition $c = y$, together with (a) allows to get prices :

$$p^\star = \frac{\alpha m_0}{(1-\alpha)y_0} \qquad (b)$$

so that

$$w^\star = \beta\ell_0^{\beta-1} \times \frac{\alpha m_0}{(1-\alpha)y_0} = \frac{\alpha\beta}{1-\alpha}\mathbf{m_0}\ell_0^{-1}$$

- Note that the way the model is solved is similar to solving the AD-AS model.

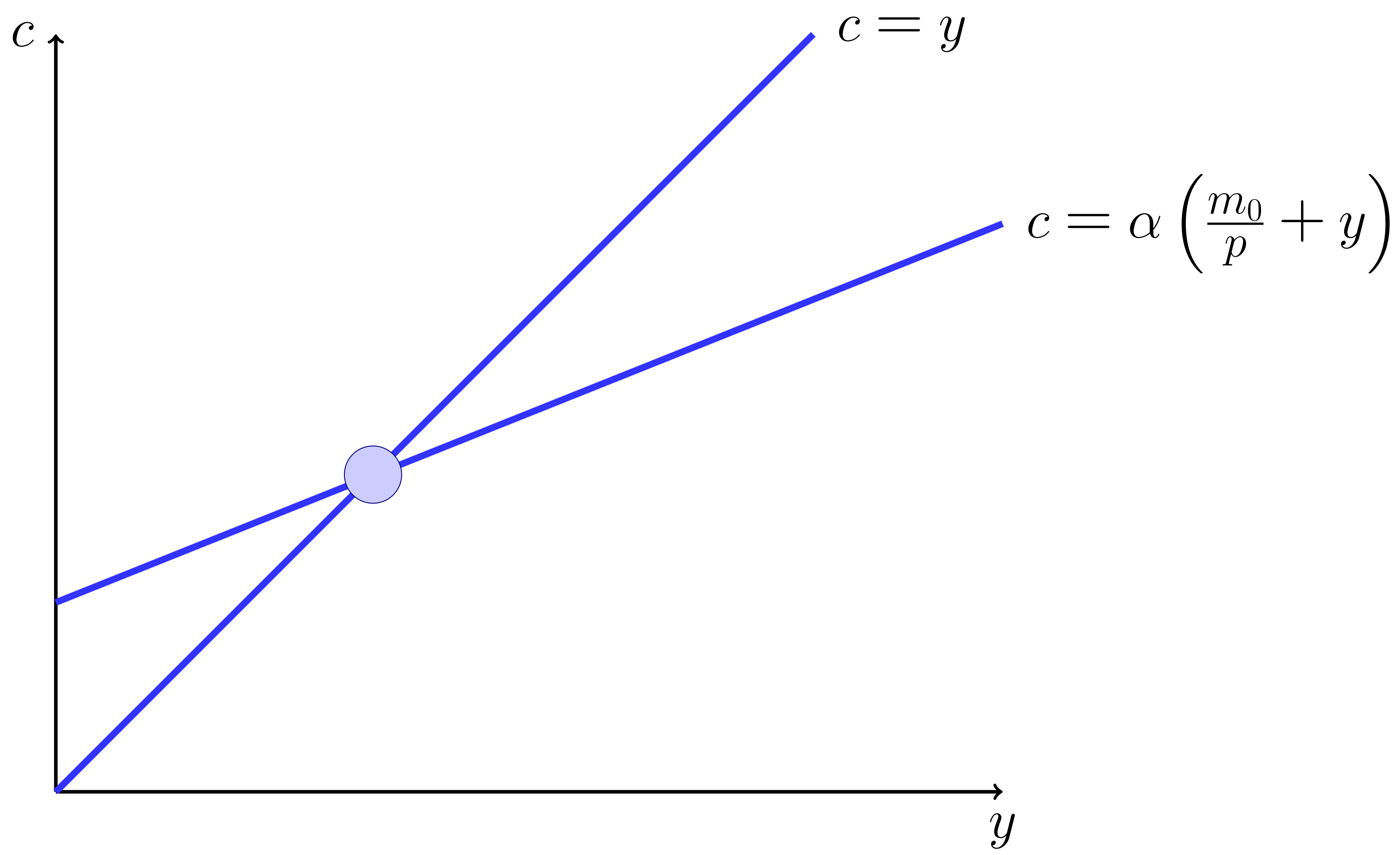
2.3 A Graphical Interpretation

- The model equilibrium (y, ℓ, m, p, w) can be obtained solving for an IS curve, then a AD-AS model.
- The five equations we use are

$$\begin{aligned} c &= \alpha\left(\frac{m_0}{p} + y\right) \quad \& \quad c = y && (a) \\ y &= \left(\frac{1}{\beta} \times \frac{w}{p}\right)^{\frac{-\beta}{1-\beta}} && (c) \\ \ell^d &= \left(\frac{1}{\beta} \times \frac{w}{p}\right)^{\frac{-1}{1-\beta}} && (d) \\ \ell^s &= \ell_0 && (e) \end{aligned}$$

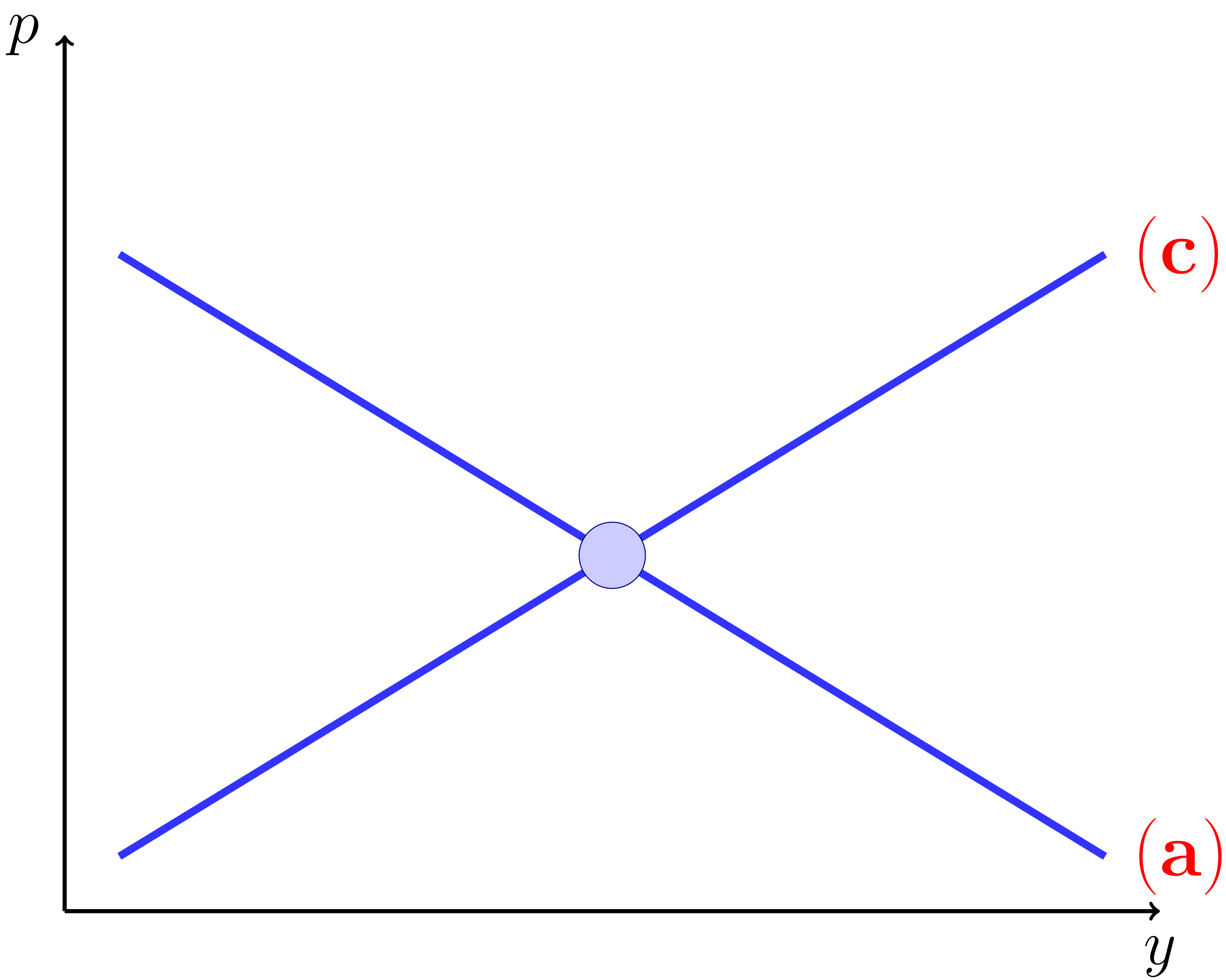
- For given p, (a) is an IS curve (planned exp. c equal to actual ones y).

Figure 1: The IS Curve (equations (a)



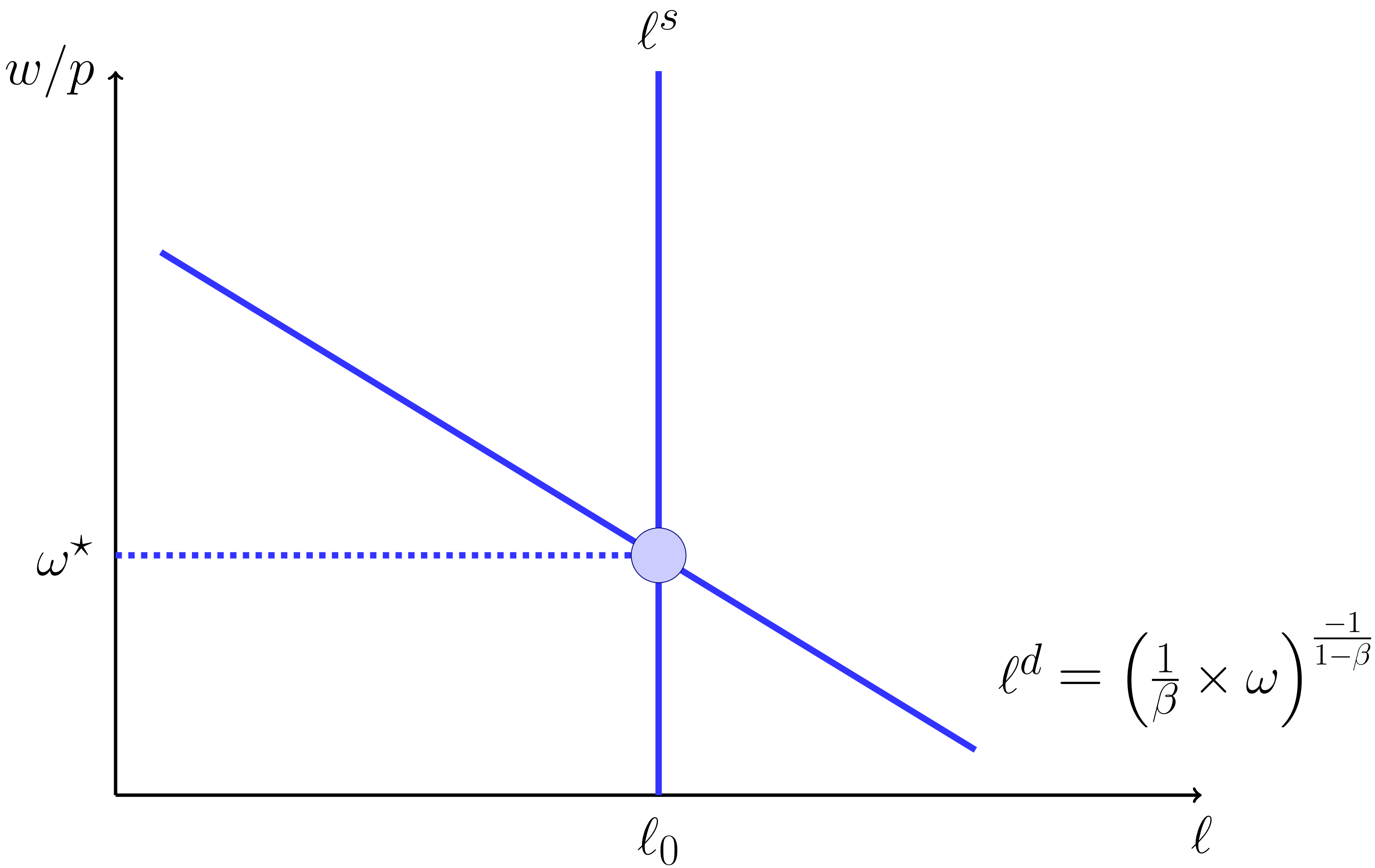
- When p varies, (a) describes an AD curve (no need here to take into account the LM curve (labor market equilibrium by Walras law) (and no bond market here)
- For a given wage, (a) is AD and (d) AS $\rightsquigarrow$ one determines y and p

Figure 2: AD-AS Given w



- Then we determine the real wage ω (and the nominal one given p) that clears the labor market, where labor demand is given by (d) and labor supply by (e)

Figure 3: Labor Market



- There is no causality in that description, as all markets clear simultaneously. One could tell a different story: labor market gives w/p and ℓ, then (c) gives y, then (a) gives y.

2.4 Transactions Outside Walrasian Equilibrium

- What does happen on a market when the price is not the equilibrium one.
- One needs to distinguish supply, demand and transactions (what is effectively traded on the market)
- Some more institutional framework is needed (rationing schemes)
- *Voluntary Exchange* is one : no one is forced to buy or sell.

2.5 Classical View of Unemployment: High Real Wages

• Assume that ω is set (rigid) above $\omega^\star$, ($\omega = \overline{\omega}$) and that voluntary exchange prevails.

• Firms: Max $\ell^\beta - \overline{\omega}\ell \leadsto \ell^d = \left(\frac{1}{\beta} \times \overline{\omega}\right)^{\frac{-1}{1-\beta}}$

• Then employment (transactions on the labor market) will be given by $\min(\ell^s, \ell^d) = \min(\ell_0, \ell^d) = \ell^d = \left(\frac{1}{\beta} \times \overline{\omega}\right)^{\frac{-1}{1-\beta}} = \overline{\ell}$

- Households are then constrained on their labor supply, and now solve:

$$\begin{aligned} \max_{c,m,\ell} \quad & \alpha \log(c) + (1-\alpha)\log(m/p) \\ st \quad & w\ell + \pi + m_0 \leq pc + m \\ & \ell \leq \overline{\ell} \end{aligned}$$

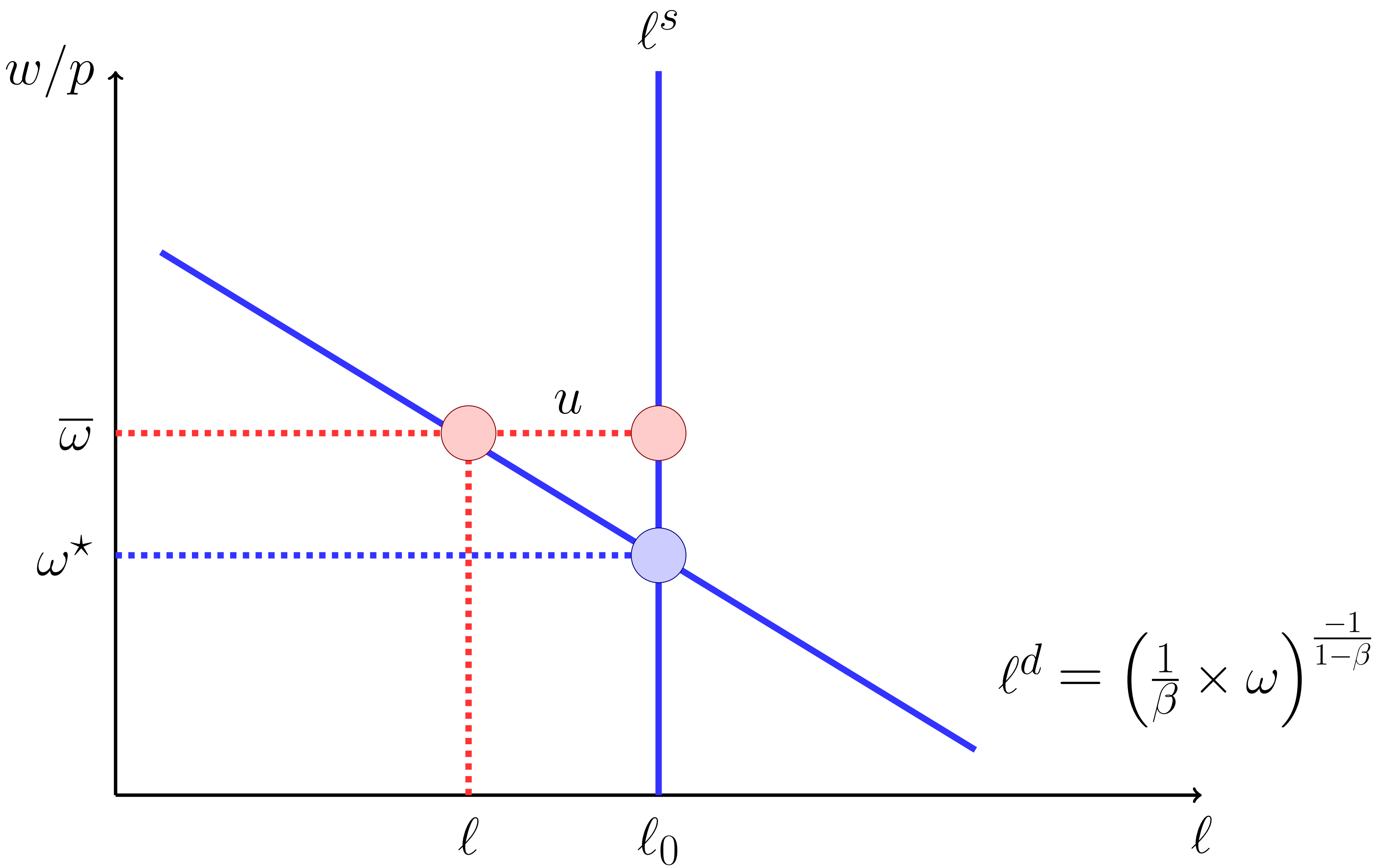
- The solution for the consumption function is $c = \alpha\left(\frac{m_0}{p} + \frac{\overline{w\ell}+\pi}{p}\right) = \alpha\left(\frac{m_0}{p} + \overline{y}\right)$
- Then p will adj ust such that $c = \overline{y}$:

$$\overline{p} = \frac{\alpha m_0}{(1-\alpha)\overline{y}}$$

and $\overline{w} = \overline{\omega} \times \overline{p}$

- This is the view according to which high real wages are the cause of unemployment, and that the problem comes from the labor market.

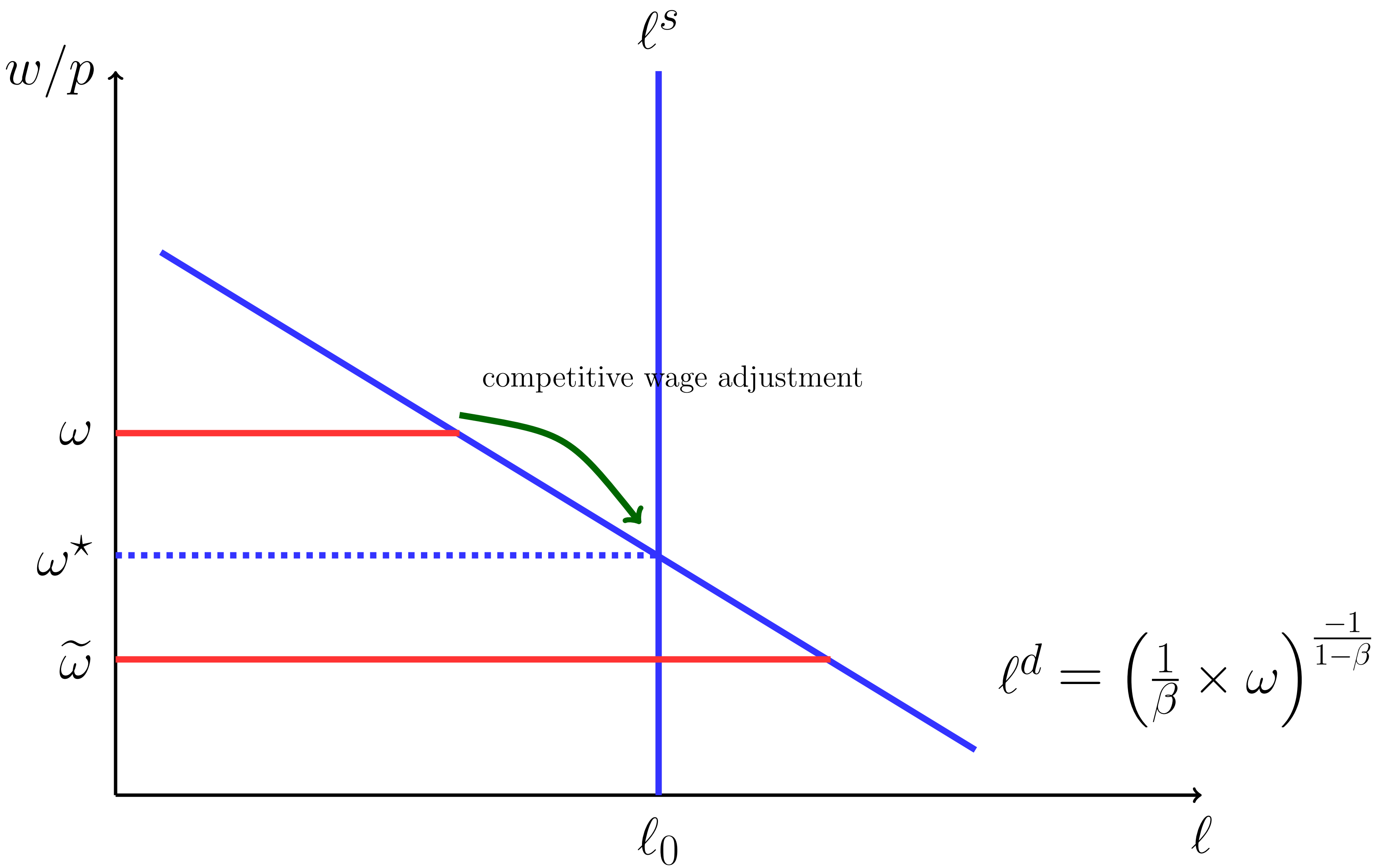
Figure 4: Classical Unemployment



2.6 A Different (Keynesian) View of Unemployment: Low Real Wages

- Assume that prices are rigid, and set to $\widetilde{p} > p^{\star}$.
- Consider a nominal wage $\widetilde{w}$ such that $\widetilde{w}/\widetilde{p} = \widetilde{\omega} \leq \omega^{\star}$
- Assume that the nominal wage does not decrease if there is no unemployment (if it were not the case, excess supply on the labor market would drive the wage down until the equality holds)

Figure 5: Labor Market with Rigid Price



- Households solve:

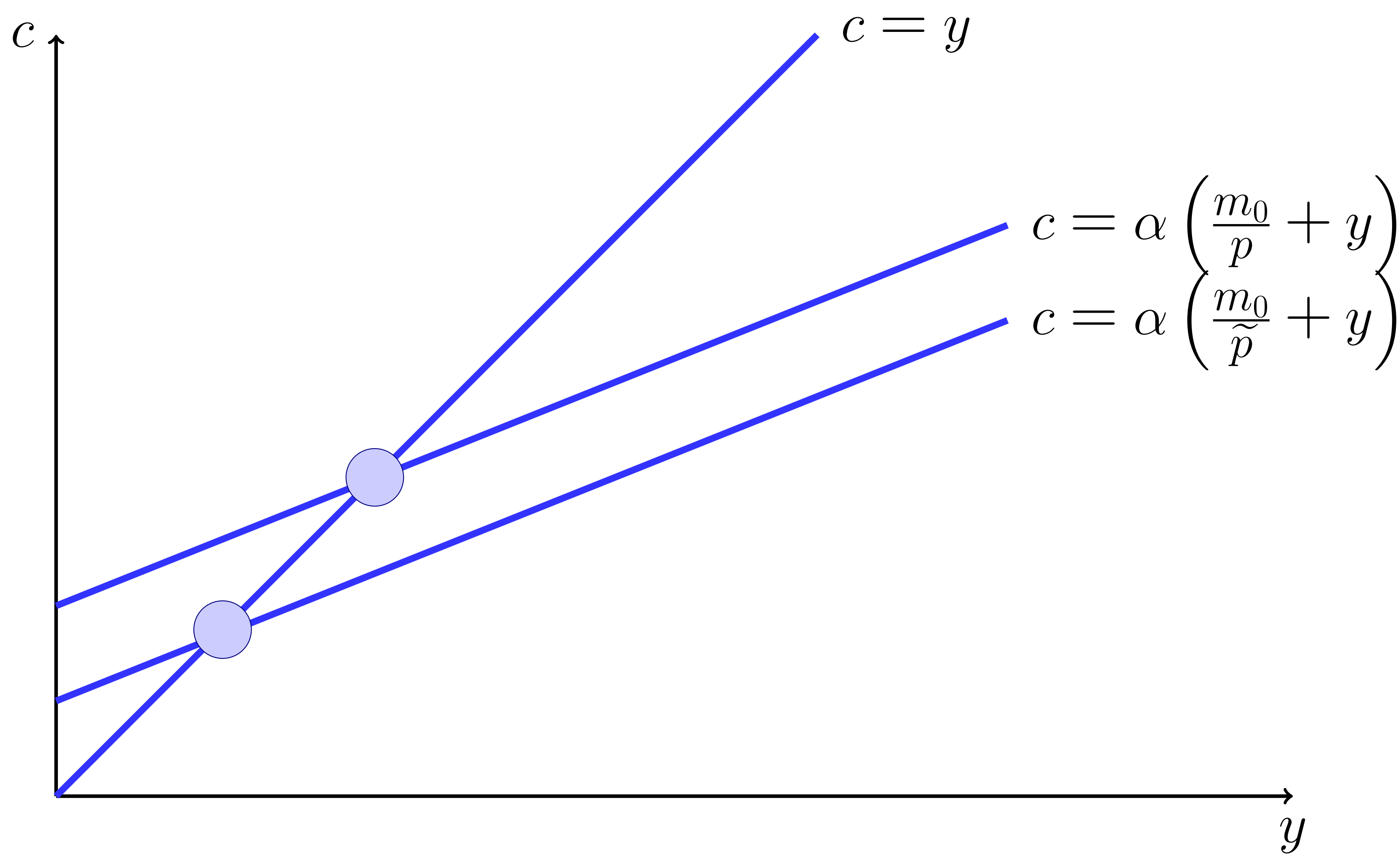
$$\begin{aligned} \max_{c,m,\ell} \quad & \alpha \log(c) + (1-\alpha)\log(m/\widetilde{p}) \\ st \quad & w\ell + \pi + m_0 \leq \widetilde{\mathbf{p}} c + m \\ & \ell \leq \widetilde{\ell} \end{aligned}$$

where $\widetilde{\ell}$ has to be determined.

- At this price, household are expressing a demand $c = \alpha\left(\frac{m_0}{\widetilde{p}} + \widetilde{y}\right)$ with $\widetilde{y} = \widetilde{\ell}^{\beta}$.

- From *(a)*, we see that c is then smaller than $y^\star$ if $\widetilde{p} > p^\star$ (would be true even in the case where the household would receive an full-employment income y_0)

Figure 6: Aggregate Demand



- We therefore we have at equilibrium of the good market

$$\widetilde{c} = \widetilde{y} = \frac{\alpha m_0}{(1-\alpha)\widetilde{\mathbf{p}}} \; < \; y^\star$$

- Firms are therefore constrained on their sales and solve: Max $\ell^\beta - \widetilde{\omega}\ell$ subject $\ell \leq \widetilde{y}^{1/\beta}$.
- It is not optimal for the firm to demand the full employment quantity of labor $\ell^\star = \ell_0$.
- Rather, the firm will limit production to the level demanded $\widetilde{y}$, and therefore hires $\widetilde{\ell} = \widetilde{y}^{1/\beta}$.
- Therefore, employment ℓ is below the full employment level ℓ_0, although the real wage is lower than its walrasian level $\rightsquigarrow$ here the

root of unemployment is the malfunctioning of the good market

$\rightsquigarrow$ importance of markets interactions and general equilibrium.

3 The Lucas Critique

- Here I want to show that if expectations are not properly taken into account, models predictions about the effect of economic policy can be misleading.
- This is the Lucas critique, that shows that estimated parameters of models where expectations are not properly modelled are not structural ones.

3.1 A Simple Model

- The model is given by two equations.
- Private agents behavior

$$n_t = \alpha c_t + \beta g_t \qquad (aa)$$

with $\alpha > 0$ and $\beta > 0$. n is employment, c is consumption, g is govt expenditures

$$c_t = \gamma n^e_{t+1}, \qquad \gamma > 0$$

where n^e_{t+1} is the expectation of period $t+1$ employment based on the information of period t (the variables dated t + the knowledge of the model equations, parameters and the process of the government spending shocks).

- Economic policy

$$g_t = \rho g_{t-1} + \varepsilon_t \qquad (bb)$$

with $0 < \rho < 1$ and ε *iid* with zero mean.

- The model reduces to

$$n_t = \alpha\gamma n^e_{t+1} + \beta g_t$$

$$g_t = \rho g_{t-1} + \varepsilon_t \qquad (bb)$$

- We assume $\alpha\gamma < 1$ (consumption does not overreact to future employment, neither employment does to current consumption)

- We want to compute the IRF of the economy (employment) to a government spending shock.
- In order to solve the model, one needs to specify the formation of expectations

3.2 The Solution with Naive Expectations

- Assume that $n^e_{t+1} = n_t$.
- Then the model solution is

$$n_t = \frac{\beta}{1 - \alpha\gamma} g_t$$

- The instantaneous multiplier is $\mu^N = \frac{\beta}{1-\alpha\gamma}$. It does not depend on ρ.
- Assume $g_{-1} = 0$. Then for a policy shock $\varepsilon_1 = 1$ and $\varepsilon_t = 0$ for $t > 1$, the economic impact of the policy is depicted on figures 1 and 2, together with the impact of a change in the policy rule ρ.

Figure 7: An Economic Policy Shock and the Dynamic Multiplier with Naive Expectations

- Note that n^e_{t+1} is always different from n_{t+1} (except asymptotically)
- Agents are always wrong in their forecast (except in the very

long run) in a predictable way $\rightsquigarrow$ if they were econometricians, they would eventually realize it.

3.3 The Solution with Rational Expectations

• Let us now assume that agents form rational expectations, ie with the knowledge of the true model and conditionally on the information available at t: $n^e_{t+1} = E_t n_{t+1}$ where E_t is the conditional mathematical expectation.

• The expectation is now endogenous and to solve the model, we first need to solve for the expectation.

Solving for the expectations : From *(aa)*,

$$n_{t+1} = \alpha\gamma E_{t+1} n_{t+2} + \beta g_{t+1} \qquad (cc)$$

$$E_t n_{t+1} = \alpha\gamma E_t E_{t+1} n_{t+2} + \beta E_t g_{t+1}$$

Using the Law of Iterated Expectations and *(bb)*,

$$E_t n_{t+1} = \alpha\gamma E_t n_{t+2} + \beta\rho g_t$$

- Repeating this calculation gives

$$E_t n_{t+n} = \alpha\gamma E_t n_{t+n+1} + \beta\rho^n g_t$$

and plugging into *(cc)*

$$\begin{aligned} E_t n_{t+1} = & (\alpha\gamma)^{\mathbf{n}} E_t n_{t+n+1} \\ & + \beta\left((\alpha\gamma)^{n-1}\rho^n + (\alpha\gamma)^{n-2}\rho^{n-1} + \cdots + \alpha\gamma\rho^2 + \rho\right) g_t \end{aligned}$$

Solving for n_t Let us now plug the expression of $E_t n_{t+1}$ into (aa)

$$\begin{aligned} n_t = & (\alpha\gamma)^{n+1} E_t n_{t+n+1} \\ & + \beta((\alpha\gamma)^n \rho^n + (\alpha\gamma)^{n-1} \rho^{n-1} + \cdots + (\alpha\gamma)^2 \rho^2 + \alpha\gamma\rho + 1) g_t \end{aligned}$$

- Taking the limit when n goes to infinity, one gets

$$n_t = \frac{\beta}{1 - \alpha\gamma\rho} g_t$$

- The instantaneous multiplier is now $\mu^{RE} = \frac{\beta}{1-\alpha\gamma\rho}$.
- Note the difference with the naive expectations case. Now ρ enters in the value of the multiplier (the multiplier is not a structural (deep, invariant) parameter.

3.4 The Lucas Critique

- Assume agents are forming rational expectations
- Assume that for the last 50 years, the govt persistence of spending has been $\rho = .95$ (government spending shocks are very persistent).
- An econometrician that would estimate the impact effect of govt spending shocks would find a multiplier $\widehat{\mu}$.
- This multiplier is indeed $\frac{\beta}{1-.95\times\alpha\gamma}$
- In order to economize on spending, the gvt can decide to reduce the persistence parameter to $\rho = .5$.

- If gvt economic advisors think that agents are not forming rational but naive expectations, they think that $\widehat{\mu}$ is equal to $\frac{\beta}{1-\alpha\gamma}$
- According to the gvt believes, The change is the policy rule ($\rho = .95$ to $\rho = .5$) will affect the persistence of the response of the economy but not its impact effect.
- In reality, μ will be affected by the change of ρ, becsause agents are rational in their expectations: μ will be reduced from $\frac{\beta}{1-.95\times\alpha\gamma}$ to $\frac{\beta}{1-.5\times\alpha\gamma}$
- μ is not a deep parameter.
- The estimated multiplier $\widehat{\mu}$ cannot be used for evaluating changes

of policy because it depends of agents reactions to this change of policy.

- Non structural econometric models cannot be used for policy evaluation: this is the LUCAS critique

4 Rational Expectations and the Ineffectiveness of Economic Policy

- Often, the consequences of a given policy depend on agents expectations about the future (example of a tax cut: transitory or permanent?).
- Let us illustrate the possible ineffectiveness of economic policy using a simple model, that is a simple version of Sargent and Wallace (1976).

4.1 The model

- We assume a AS-AD model, with the so-called LUCAS' suply function

$$\begin{aligned} y_t &= \lambda y_{t-1} + \alpha(p_t - p_t^e) \quad (AS) \\ y_t &= -\beta p_t + \gamma m_t \quad (AD) \end{aligned}$$

- m_t is observed in period t.

4.2 Static Expectations

- Assume that expectations are given by

$$p_t^e = p_{t-1}$$

• What matters for the result is that expectations are exogenously given

• Let us solve for the equilibrium. (AS) and (AD) imply

$$\lambda y_{t-1} + \alpha\big(p_t - p_t^e\big) = -\beta p_t + \gamma m_t \Leftrightarrow$$

$$p_t^{\star} = \frac{\gamma}{\alpha+\beta} m_t - \frac{\lambda}{\alpha+\beta} y_{t-1} + \frac{\alpha}{\alpha+\beta} p_{t-1}$$

and plugging into (AD) yields

$$y_t^{\star} = \gamma\left(1 - \frac{\beta}{\alpha+\beta}\right) m_t + \frac{\beta\lambda}{\alpha+\beta} y_{t-1} - \frac{\alpha\beta}{\alpha+\beta} p_{t-1}$$

• We are in a "keynesian type " AS-AD model,with non vertical

AS, and the monetary multiplier is $\frac{dy_t^\star}{dm_t} = \gamma\left(1 - \frac{\beta}{\alpha+\beta}\right) > 0$.

4.3 The Rational Expectations Equilibrium

- We now assume that agents fully know and understand the economic model, and therefore form rational expectations: $p_t^e = E_{t-1}p_t$ where E is the mathematical expectation, conditional on the knowledge of the model.
- To solve the model, we need to proceed in two steps: fist compute the equilibrium *in expected terms*, to compute the equilibrium value of the *endogenous* variable $E_{t-1}p_t$, and then solve for

the actual equilibrium given the equilibrium value of $E_{t-1}p_t$.

Expectation computation : Let's write the model equilibrium in expected terms

$$E_{t-1}\left(\lambda y_{t-1} + \alpha\big(p_t - E_{t-1}p_t\big)\right) = E_{t-1}\left(-\beta p_t + \gamma m_t\right) \Leftrightarrow$$

$$E_{t-1}p_t = \frac{\gamma}{\beta}E_{t-1}m_t - \frac{\lambda}{\beta}y_{t-1}$$

Solving for the equilibrium : Using the expression of the expectation, the AS curve becomes

$$y_t = \lambda y_{t-1} + \alpha p_t - \alpha\gamma/\beta E_{t-1}m_t + \alpha\lambda/\beta y_{t-1} \qquad (\star)$$

From AS, one has

$$p_t = \frac{\gamma}{\beta} m_t - \frac{1}{\beta} y_t$$

Plugging into $(\star)$,

$$y_t = \lambda y_{t-1} + \alpha\gamma/\beta m_t - \frac{\alpha}{\beta} y_t - \alpha\gamma/\beta E_{t-1} m_t + \alpha\lambda/\beta y_{t-1}$$

that gives

$$y_t = \frac{\alpha\gamma}{\alpha+\beta} \underbrace{\left(m_t - E_{t-1} m_t\right)}_{\text{surprise}} + \lambda y_{t-1}$$

4.4 Comments

- Anticipated monetary policy is inefficient $\rightsquigarrow$ the AS curve is vertical on average
- Only monetary surprises are efficient $\rightsquigarrow$ non systematic effect of monetary policy

4.5 Inefficiency of a feedback rule

- Assume a feedback rule of the type $m_t = -\zeta y_{t-1}$.
- If output was below the non-stochastic equilibrium level in period $t-1$ (which means that $y_{t-1} < 0$), then monetary policy is

expansionary in t.

- It is easy to check that $m_t - E_{t-1}m_t = 0 \rightsquigarrow$ the policy is inefficient.

5 Ricardian Equivalence

- Key idea: the timing of lump taxes does not matter $\rightsquigarrow$ equivalence of the debt/lump taxes timing
- This is the equivalent in macro of the Modigliani-Miller theorem.
- Formally presented by Barro, JPE, 1974
- It means that the "keynesian multiplier $\Delta T = \Delta B/P$" is fallacious.
- I present the model, then state the result and discuss it.

5.1 An Infinitely Lived-Agent Economy

5.1.1 The Setting

- N identical households

$$\sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

with all good properties, including $\lim_{c \downarrow 0} u'(c) = +\infty$

- No uncertainty
- The household can invest in a single risk-free asset bearing a fixed gross one-period rate of return $R > 1$: it is a loan to foreigners or to the government.

- 1 unit of b_{t+1} is a piece of paper that is sold R^{-1} units of good in period t and that promises 1 units of good in $t+1$.
- $b > 0$ means that the Hh is net creditor, $b < 0$ net borrower.
- The time t budget constraint (BC) is

$$c_t + R^{-1} b_{t+1} \leq y_t + b_t \tag{2}$$

with b_0 given.

- Assume that $R\beta = 1$ and that $\{y_t\}_{t=0}^{\infty}$ is a given nonstochastic nonnegative endowment sequence with $\sum_{t=0}^{\infty} R^{-t} y_t < \infty$.
- The extent to which Ricardian Equivalence holds depends on

households' access to financial markets. We explore two possibilities.

- The first one is that the household can lend but not borrow: $b_t \geq 0$ for all t.
- The second one is that the household cannot borrow more that it is feasible to repay: $b_t \geq \widetilde{b}_t$ for all t.
- I will (loosely) refer to this case as the no financial constraint case.
- This maximum amount **(in absolute terms)** $\widetilde{b}_t$ is computed

by setting $c_t = 0$ for all t in (2) and solving forward:

$$\widetilde{b}_t = -\sum_{j=0}^{\infty} R^{-j} y_{t+j} \tag{3}$$

where the following transversality condition have been imposed:

$$\lim_{T \to \infty} R^{-T} b_T = 0 \tag{4}$$

- This $\widetilde{b}_t$ is referred to as the *natural debt limit* and the alternative restriction is

$$b_t \geq \widetilde{b}_t \tag{5}$$

5.1.2 Solution to Consumption/Saving Decision in the No Financial Constraint Case

• The the typical intertemporal household problem is here to maximize (1) s.t. (2).

$$\max_{b_{t+1},c_t} \mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + \lambda_t (y_t + b_t - c_t - R^{-1} b_{t+1}) \right]$$

• The FOC are

$$\begin{cases} u'(c_t) = \lambda_t \\ R^{-1}\lambda_t = \beta\lambda_{t+1} \\ \lambda_t(y_t + b_t - c_t - R^{-1}b_{t+1}) = 0 \\ \lambda_t \geq 0 \end{cases}$$

- which implies:

$$u'(c_t) = \beta R u'(c_{t+1}) \quad \forall t \geq 0$$

5.1.3 Solution to Consumption/Saving Decision in the $b_t \geq 0$ Case

- The the typical intertemporal household problem is here to maximize (1) s.t. (2) and $b_{t+1} \geq 0$.

$$\max_{b_{t+1},c_t} \mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + \lambda_t (y_t + b_t - c_t - R^{-1} b_{t+1}) + \mu_t b_{t+1} \right]$$

- The FOC are

$$\begin{cases} u'(c_t) = \lambda_t \\ R^{-1}\lambda_t - \mu_t = \beta\lambda_{t+1} \\ \lambda_t\left(y_t + b_t - c_t - R^{-1}b_{t+1}\right) = 0 \\ \mu_t b_{t+1} = 0 \\ \lambda_t \geq 0 \\ \mu_t \geq 0 \end{cases}$$

- which gives

$$u'(c_t) \geq \beta R u'(c_{t+1}) \quad \forall t \geq 0 \tag{6a}$$

$$u'(c_t) > \beta R u'(c_{t+1}) \quad \text{implies} \quad b_{t+1} = 0 \tag{6b}$$

- with $\beta R = 1$, this becomes $c_t = c_{t+1}$ when the consumer is not constrained ($b_{t+1} \geq 0$) and $c_{t+1} > c_t = y_t + b_t$ when she is constrained ($b_{t+1} = 0$).

Example 1 : $b_0 = 0$, $\{y_t\}_{t=0}^{\infty} = \{y_h, y_l, y_h, y_l, ...\}$ with $y_h > y_l > 0$, $b_t \geq 0 \ \forall t$.

Example 2 : $b_0 = 0$, $\{y_t\}_{t=0}^{\infty} = \{y_l, y_h, y_l, y_h, ...\}$ with $y_h > y_l > 0$, **with** $b_t \geq 0$ **or with** $b_t \geq \widetilde{b}_t$

Example 3 : $b_0 = 0$, $y_t = \lambda^t$ where $1 < \lambda < R$ **with** $b_t \geq 0$

Example 4 : $b_0 = 0$, $y_t = \lambda^t$ where $1 < \lambda < R$ and $b_t \geq \widetilde{b}_t$ is imposed.

Example 5 : $b_0 = 0$, $y_t = \lambda^t$ where $1 > \lambda$ and $b_t \geq \widetilde{b}_t$ is imposed.

5.2 Introducing a Government and Stating Ricardian Equivalence

5.2.1 The Government

- The Gvt purchases a stream $\{g_t\}_{t=0}^{\infty}$ per household, imposes a stream of lump-sum taxes $\{\tau_t\}_{t=0}^{\infty}$ and is subject to the BC:

$$B_t + g_t = \tau_t + R^{-1}B_{t+1} \tag{8}$$

- B_t is a one-period debt due at t and denominated in period t consumption good. The Gvt is allowed to borrow.
- We impose the transversality condition $\lim_{T\to\infty} R^{-T}B_{T+1} = 0$.

- One gets from solving (8) forward:

$$B_t = \sum_{j=0}^{\infty} R^{-j} \left(\tau_{t+j} - g_{t+j}\right) \tag{9}$$

5.2.2 Households

- The household's BC (2) becomes

$$c_t + R^{-1} b_{t+1} \leq y_t - \tau_t + b_t \tag{10}$$

Solving forward and using the transversality condition:

$$b_t = \sum_{j=0}^{\infty} R^{-j} (c_{t+j} + \tau_{t+j} - y_{t+j}) \tag{11}$$

and the natural debt limit is

$$\widetilde{b}_t = \sum_{j=0}^{\infty} R^{-j}(\tau_{t+j} - y_{t+j}) \tag{12}$$

- Note that the debt limit is greater (I mean more binding) with positive taxes.

5.2.3 Equilibrium and Ricardian Equivalence

DEFINITION 2 *Given initial condition* (b_0, B_0)*, an* **equilibrium** *is a household plan* $\{c_t, b_{t+1}\}$ *and a government policy* $\{g_t, \tau_t, B_{t+1}\}$ *such that (a) the government plan satisfies the government BC (8) and (b) given* $\{\tau_t\}$*, the household plan is optimal.*

Proposition 1 **Ricardian Equivalence :** *Suppose that the natural debt limit prevail. Given initial conditions* (b_0, B_0)*, let* $\{\overline{c}, \overline{b}_{t+1}\}$ *and* $\{\overline{g}_t, \overline{\tau}_t, \overline{B}_{t+1}\}$ *be an equilibrium. Consider any other tax policy* $\{\widehat{\tau}_t\}$ *satisfying*

$$\sum_{t=0}^{\infty} R^{-t}\widehat{\tau}_t = \sum_{t=0}^{\infty} R^{-t}\overline{\tau}_t \tag{13}$$

Then $\{\overline{c}_t, \widehat{b}_{t+1}\}$ *and* $\{\overline{g}_t, \widehat{\tau}_t, \widehat{B}_{t+1}\}$ *is also an equilibrium where*

$$\widehat{b}_t = \sum_{j=0}^{\infty} R^{-j}\left(\overline{c}_{t+j} + \widehat{\tau}_{t+j} - y_{t+j}\right) \tag{14}$$

and

$$\widehat{B}_t = \sum_{j=0}^{\infty} R^{-t} \left(\widehat{\tau}_{t+j} - \overline{g}_{t+j} \right) \tag{15}$$

• In words, the timing of taxes and debt does not matter. What matters is their present value.

Proof of the proposition : We need to show *(i)* that the consumption plan $\{\overline{c}_t\}$ and the adjusted borrowing plan $\{\widehat{b}_t\}$ solve the household's optimum problem and *(ii)* that the altered government tax and borrowing plans continue to satisfy the government's BC.

$\looparrowright$ *(i)* At time 0, the household face a single intertemporal budget constraint (this is true under the natural debt limit)

$$b_0 = \sum_{t=0}^{\infty} R^{-t}(c_t - y_t) + \sum_{t=0}^{\infty} R^{-t}\tau_t$$

Therefore, the household's optimal consumption plan does not depend on the timing of taxes, but only on their net present value $\rightsquigarrow$ $\{\bar{c}_t\}$ is still feasible and optimal.

Having $\{\bar{c}_t\}$, we can construct the sequence of $\{\widehat{b}_{t+1}\}$ by solving the household 's BC (10) forward to obtain (14). To do so, we use a transversality condition $\lim_{T\to\infty} R^{-T}\widehat{b}_{T+1} = 0$. Let's check that it is satisfied if the transversality condition is satisfied for the original borrowing plan:

In an period $k-1$, solving the BC (10) backwards yields

$$b_k = \sum_{j=1}^{k} R^j \left(y_{k-j} - \tau_{k-j} - c_{k-j}\right) + R^k b_0$$

which gives

$$\overline{b}_k - \widehat{b}_k = \sum_{j=1}^{k} R^j \left(\widehat{\tau}_{k-j} - \overline{\tau}_{k-j}\right)$$

which is also, by $\times R^{1-k}$

$$R^{1-k}\left(\overline{b}_k - \widehat{b}_k\right) = R \sum_{t=0}^{k-1} R^{-t} \left(\widehat{\tau}_t - \overline{\tau}_t\right)$$

The limit of the RHS is zero when $k \to \infty$ because of (13). Then, given that $\{\bar{b}_{t+1}\}$ satisfies the TC, $\{\widehat{b}_{t+1}\}$ does.

↬ *(ii)* Let us show now that the altered government tax and borrowing plans satisfy the government BC. This BC is given by

$$B_0 = \sum_{t=0}^{\infty} R^{-j} \tau_t - \sum_{t=0}^{\infty} R^{-j} g_t$$

From (13), we now that the BC is still satisfied. The sequence of $\widehat{B}_{t+1}$ can then be recovered by solving forward this BC at every period t. □

5.2.4 The No-Borrowing Constraint Case

- The former proposition relies on the fact that the household can undo what the government does by using financial markets.
- This neutrality results does not hold any more in the no-borrowing constraint case.
- Now, a change in the timing of taxes can cause a previously non binding constraint binding.
- We have only a weak form of neutrality

PROPOSITION 2 **A weak form of Ricardian Equivalence :**

Consider an initial equilibrium with consumption path $\{\bar{c}\}$ *in which* $b_{t+1} > 0$ *for all* $t \geq 0$*. Let* $\{\bar{\tau}_t\}$ *be the tax rate in the initial equilibrium, and let* $\{\widehat{\tau}_t\}$ *be any other tax rate sequence with same present value and for which*

$$\widehat{b}_t = \sum_{j=0}^{\infty} (\bar{c}_{t+j} + \widehat{\tau}_{t+j} - y_{t+j}) \geq 0 \qquad (\star)$$

for all $t \geq 0$*. Then* $\{\bar{c}_t\}$ *is also an equilibrium allocation for the* $\{\widehat{\tau}_t\}$ *sequence.*

Proof : If $(\star)$ is satisfied, then the household can undo the change in the government tax and borrowing plan without hitting the no-borrowing constraint. The sequence $\{\bar{c}_t\}$ is therefore feasible, and then, one can proceed as in the preceding proof. □

5.3 A Linked Generations Interpretation

- Often the Ricardian equivalence results is dismissed as irrealistic because the time horizon of some households is shorter than the government one ("I'll be dead before they start raising taxes to pay back public debt $\leadsto$ for me, government bonds are net wealth")
- Barro was the first to show that this is not true if generations

are linked by bequests.

• The model with borrowing constraints can be reinterpreted in such a way:

• Assume that there is a sequence of one-period-lived agents, that value consumption and the utility of its unique offspring:

$$u(c_t) + \beta V(b_{t+1})$$

where b_{t+1} is the amount of bequest that is left to generation $t+1$ and V is the maximized utility of a time $t+1$ agent, recursively defined as

$$V(b_t) = \max_{c_t, b_{t+1}} \{u(c_t) + \beta V(b_{t+1})\} \quad (16)$$

$$\text{s.t. } c_t + R^{-1} b_{t+1} \leq y_t - \tau_t + b_t \quad (17)$$

with $b_{t+1} \geq 0$

- This model consumption equilibrium allocations are identical to those of the infinitely-lived one with a no-borrowing constraint. Therefore, the weak version of the Ricardian Equivalence theorem holds.

5.4 Reasons for Which the Ricardian Equivalence Theorem Might not Hold

5.4.1 Intergenerational Redistribution

- As I said before, if tax cut or the current generation are financed by tax increase on the next generation, Ricardian Equivalence does not hold
- This is not true if there is intergenerational altruism.

5.4.2 Capital Market Imperfections

• Again, I have shown before that only a weak form of Ricardian Equivalence holds if there is a no-borrowing constraint.

• It is also the case if there is a wedge between creditor's interest rate and debtor's one.

5.4.3 Distortionary Taxes

• If taxes are distortionary, then their timing affect household's decisions.

5.4.4 Income Uncertainty

- Government debt might affect consumer's perception of the risks they face, and therefore affects their current consumption.
- Assume that taxes are levied as a function of income, and that future income is uncertain.
- Assume that the government cuts taxes today, issues debt today and raises income taxes in the future to pay off the debt.
- In such a case, consumer's expected lifetime income is unchanged, but the uncertainty they face is reduced. If the have

precautionary savings, this reduction in uncertainty will reduce those savings and therefore foster consumption.

5.5 Empirical Issues

- Difficult to test directly. The Ricardian argument does not render all fiscal policy irrelevant.
- For example, if the government cut taxes today and households expect this tax cut to be met with future cuts in useless government expenditures, households' permanent income increases and so does consumption. $\rightsquigarrow$ but one does not observe directly

expectations...

• Some assumptions or implications of the Ricardian Equivalence result can be tested.

5.5.1 Testing Assumptions

• It has been shown that consumers do not smooth consumption as much as Permanent Income theory predicts $\rightsquigarrow$ there are liquidity constraints, financial imperfections,...

5.5.2 Testing Implications for Consumption

- In a consumption equation

$$C = f(\text{income, wealth, fiscal policy, taxes, public debt,...})$$

the coefficients on taxes and public debt should be zero.

- but a lot of implementation problems (expectations (suppose that the current level of taxes affect expectations about future government expenditures) , simultaneity (shocks to consumption might affect fiscal policy),...)