Output dynamics in RBC Models

Cogley and Nason (1995)

Alessandro Ispano, Peter Schwardmann and Damian Tago

MACRO II

February 2010

The paper in one slide

- Central question of the paper: Are RBC models consistent with the observed output dynamics? (Or: How often would an econometrician observe the same kind of stylized facts in data generated by RBC models?)

The paper in one slide

- Central question of the paper: Are RBC models consistent with the observed output dynamics? (Or: How often would an econometrician observe the same kind of stylized facts in data generated by RBC models?)
- Methodology: generate artificial data by simulating RBC models and test their consistency with real data.

The paper in one slide

- Central question of the paper: Are RBC models consistent with the observed output dynamics? (Or: How often would an econometrician observe the same kind of stylized facts in data generated by RBC models?)
- Methodology: generate artificial data by simulating RBC models and test their consistency with real data.
- Conclusion: in physics, it takes 3 laws to explain 99% of the data; in economics, it takes more than 99 models to explain about 3%.

Organization of the presentation

- **Part 1**: stylized facts about output dynamics and econometric methodology.
- **Part 2**: baseline RBC models – comparison between simulated and actual data.
- **Part 3**: extensions (gestation lags and capital adjustment costs, employment lags and labor adjustment costs) - comparison between simulated and actual data.

Stylized facts about output dynamics (1/2)

- 3 dimensions of business cycle:
  - **periodicity of output**;
  - comovements of other variables with output;
  - relative volatilities of various series.
- 2 stylized facts:
  - positive autocorrelation of output growth over short horizons;
  - trend-reverting component of output that has a hump-shaped impulse-response function.

Stylized facts about output[1] dynamics (2/2)

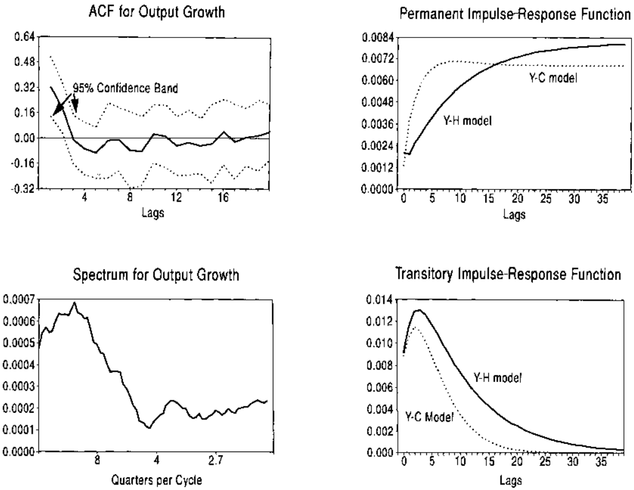


FIGURE 1. STYLIZED FACTS ABOUT GNP DYNAMICS

[1] Real per capita U.S. GNP, 1954:1-1988:4.

How are IRF estimated?

- Based on structural VAR technique by Blanchard and Quah (1989):
  - SVAR for output and unemployment;
  - estimate the reduced form and compute its moving average vectorial representation;
  - from this, recover the moving average vectorial representation of the structural form, imposing the restriction that the "demand" shock does not have an impact on output in the lung run.
- In this paper: second-order VAR for output growth and hours worked.

How to check the consistency between simulations and stylized facts

- Each RBC model is simulated 1000 times over an horizon of 140 quarters.
- Consistency is tested on 3 levels:
  - autocorrelation function of output growth;
  - spectrum for output growth;
  - impulse response functions for output.

Test for the autocorrelation function

Generalized Q-statistics:

$$Q_{acf} = (\hat{c} - c)' \hat{V}_c^{-1} (\hat{c} - c),$$

where $\hat{c}$ is the sample ACF, c is the model-generated ACF and $\hat{V}_c$ is the covariance matrix.

c and $\hat{V}_c$ are calculated as

$$c = \frac{1}{n} \sum_{i=1}^{n} c_i;$$

$$\hat{V}_c = \frac{1}{n} \sum_{i=1}^{n} (c_i - c)(c_i - c)'.$$

Q_{acf} is approximately distributed as a chi-square with degrees of freedom equal to the number of elements in c (8 in this case).

Spectrum

- The theoretical (simulated) spectrum is estimated by smoothing the ensemble averaged periodogram, together with upper and lower 2.5% probability bounds.
- Do the sample spectrum falls in these bounds?

Test for the impulse response functions

Generalized Q-statistics (as for ACF):

$$Q_{irf} = (\hat{r} - r)' \hat{V}_r^{-1} (\hat{r} - r),$$

where $\hat{r}$ and r are the sample and model-generated IRF respectively. $\hat{V}_r$ is the covariance matrix.

r and $\hat{V}_r$ are calculated as

$$r = \frac{1}{n} \sum_{i=1}^{n} r_i;$$

$$\hat{V}_r = \frac{1}{n} \sum_{i=1}^{n} (r_i - r)(r_i - r)'.$$

Q_{irf} is approximately distributed as a chi-square with degrees of freedom equal to the number of elements in r (8 in this case).

The RBC baseline model (1/4)

- RBC models rely on three propagation mechanisms:
  - capital accumulation;
  - intertemporal substitution;
  - adjustment lags and costs (next section).
- A RBC model due to Christiano and Eichenbaum (1992) is used as the baseline model.

The RBC baseline model (2/4)

- A representative consumer has the following preferences:

$$E_t \left\{ \sum_{j=0}^{\infty} \beta^j \left[\ln(c_{t+j}) + \gamma (N - n_{t+j}) \right] \right\},$$

where c_t is consumption, N is the total endowment of time, n_t are labor hours and β is the discount factor.

- A representative firm produces output with the following Cobb-Douglas production function:

$$y_t = k_t^{\theta} (a_t n_t)^{1-\theta},$$

where y_t is output, k_t is the capital stock and a_t is a technology shock.

The RBC baseline model (3/4)

- The law of motion of capital is given by:

$$k_{t+1} = (1-\delta)k_t + i_t,$$

where δ is the depreciation rate and i_t is gross investment.

- The model is driven by technology and government spending shocks:

$$(1-L)\ln(a_t) = \mu + \varepsilon_{at},$$

$$\ln(g_t) - \ln(a_t) = \bar{g} + \frac{\varepsilon_{gt}}{1-\rho L},$$

where g_t is government spending, and ε_{at} and ε_{gt} are the technology and government spending innovations, respectively.

The RBC baseline model (4/4)

- Estimates from Christiano and Eichenbaum are used to put values on the parameters of the model.
- The innovation variances are rescaled to match the sample variance of per capita output growth.
- The model has a balanced growth path.
- The log of per capita output inherits the trend properties of total factor productivity and is therefore difference stationary.

Testing the baseline model (1/3)

TABLE 1—BASELINE REAL-BUSINESS-CYCLE MODELS

Model	Q_{acf}	Q_{irf} y_P	Q_{irf} y_T
King et al. (1988b)	23.0	—	—
	(0.003)		
Hansen (1989)	20.5	—	—
	(0.008)		
Greenwood et al. (1988)	25.7	—	—
	(0.001)		
Christiano and	22.1	50.1	449.3
Eichenbaum (1992)	(0.005)	(0.018)	(0.000)
Benhabib et al. (1991)	31.2	23.0	163.9
	(0.0001)	(0.064)	(0.001)
Braun (1994)	19.9	25.2	194.3
	(0.011)	(0.054)	(0.001)

Notes: This table reports test statistics for the autocorrelation and impulse-response functions. The statistics Q_{acf} and Q_{irf} are defined by equations (6)–(8) and (9)–(11), respectively. The variable y_P refers to the permanent component of output, and y_T refers to the transitory component. The autocorrelation and impulse-response functions are truncated at lag 8, and probability values are in parentheses.

Testing the baseline model (2/3)

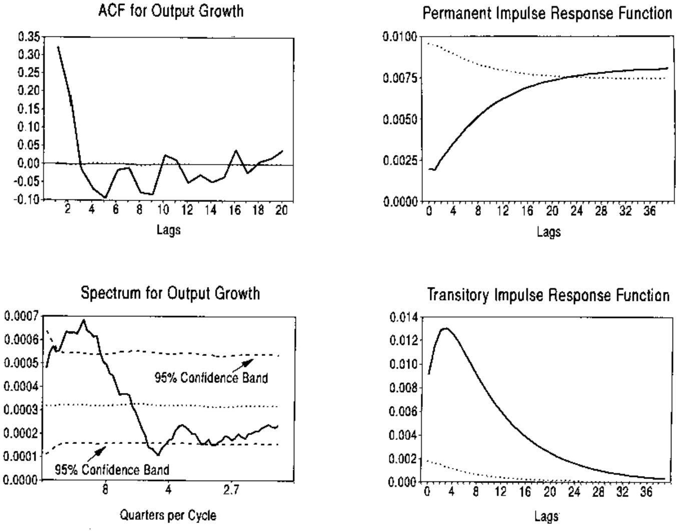


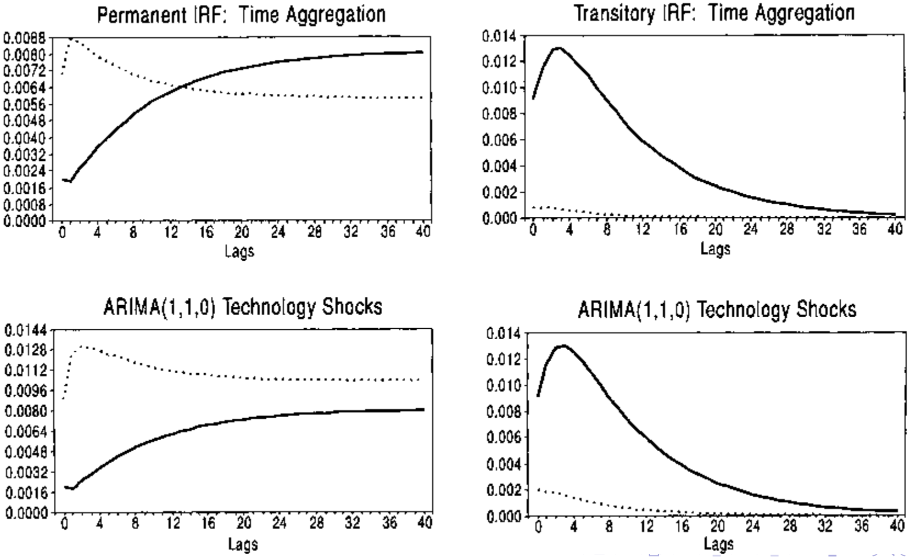
FIGURE 3. THE CHRISTIANO-EICHENBAUM MODEL

Testing the baseline model (3/3)

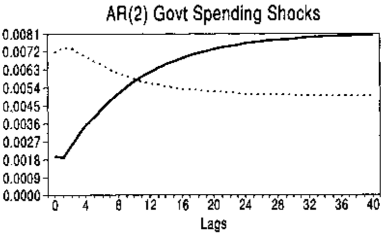
- Impulse and propagation:
  - in the model, technology shocks account for most of the variation in output growth;
  - technology shocks follow a random walk, therefore autocorrelations for growth in total factor productivity are zero;
  - autocorrelations for output growth are also close to zero. This might suggest weak propagation mechanisms;
  - the model generated impulse response functions confirm this diagnosis.

External sources of output dynamics (1/3)

- Three external sources of dynamics to compensate for weak propagation mechanisms:
  - temporal aggregation;
  - serially correlated increments to total factor productivity;
  - higher order autoregressive representations for transitory shocks.


Permanent IRF: Time Aggregation
0.0088
0.0080
0.0072
0.0064
0.0056
0.0048
0.0040
0.0032
0.0024
0.0016
0.0008
0.0000
0 4 8 12 16 20 24 28 32 36 40
Lags
Transitory IRF: Time Aggregation
0.014
0.012
0.010
0.008
0.006
0.004
0.002
0.000
0 4 8 12 16 20 24 28 32 36 40
Lags
ARIMA(1,1,0) Technology Shocks
0.0144
0.0128
0.0112
0.0096
0.0080
0.0064
0.0048
0.0032
0.0016
0.0000
0 4 8 12 16 20 24 28 32 36 40
Lags
ARIMA(1,1,0) Technology Shocks
0.014
0.012
0.010
0.008
0.006
0.004
0.002
0.000
0 4 8 12 16 20 24 28 32 36 40
Lags

External sources of output dynamics (3/3)



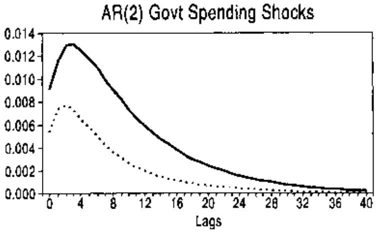


TABLE 2—EXTERNAL SOURCES OF DYNAMICS

Source	Q_{acf}	Q_{irf}: y_P	Q_{irf}: y_T
Temporal aggregation	7.7	11.6	803.7
	(0.467)	(0.182)	(0.000)
ARIMA(1,1,0)	8.6	117.7	152.1
technology shocks	(0.376)	(0.003)	(0.003)
AR(2) government	5.5	23.2	14.4
spending shocks	(0.705)	(0.070)	(0.126)

Effects of gestation lags and adjustment costs (1/6)

- Why incorporate these new features?
  - propagation of shocks (effect on investment decisions).

- Capital gestation lags:
  - time-to-build model (3-quarter gestation lag);
  - Rouwenhorst (1991).

- Capital adjustment costs:
  - q-theoretic model (quadratic costs):

$$\ln(y_t) = \ln\left(f\left(k_t, a_t, n_t\right)\right) - \left(\frac{\alpha_k}{2}\right)\left(\frac{\Delta k_t}{k_{t-1}}\right)^2.$$

Effects of gestation lags and adjustment costs (2/6)

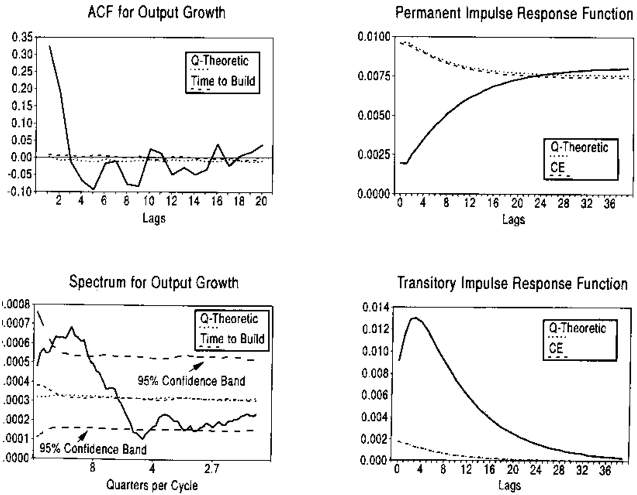


FIGURE 5. GESTATION LAGS AND CAPITAL ADJUSTMENT COSTS

Note: Solid lines show sample moments, and dashed and dotted lines show model-generated moments.

Effects of gestation lags and adjustment costs (3/6)

Gestation Lags and Capital Adjustment Costs			
		Q(irf)	
Model	Q(acf)	*y(p)*	*y(t)*
Time-to-build	20.6 (0.008)	-	-
Q-theory (2.2)	20.1 (0.008)	57.4 (0.015)	419.3 (0.000)
Sensitivity analysis			
(2.2/4)	21.8 (0.005)	40.1 (0.028)	410.8 (0.000)
(2.2/2)	21.4 (0.006)	40.0 (0.028)	406.5 (0.000)
(2*(2.2))	19.0 (0.015)	71.1 (0.008)	385.9 (0.000)
(4*(2.2))	16.4 (0.036)	95.9 (0.005)	387.1 (0.000)

Effects of gestation lags and adjustment costs (4/6)

- Employment lags:
  - labor-hoarding model: adjusting size vs adjusting effort;
  - inability of making current-quarter employment adjustments;
  - Burnside (1993).

- Labor adjustment costs:
  - quadratic costs of adjusting labor input;
  - Saphiro's estimates used for calibration.

$$\ln(y_t) = \ln\left(f\left(k_t, a_t, n_t\right)\right) - \left(\frac{\alpha_k}{2}\right)\left(\frac{\Delta k_t}{k_{t-1}}\right)^2 - \left(\frac{\alpha_n}{2}\right)\left(\frac{\Delta n_t}{n_{t-1}}\right)^2.$$

Effects of gestation lags and adjustment costs (5/6)

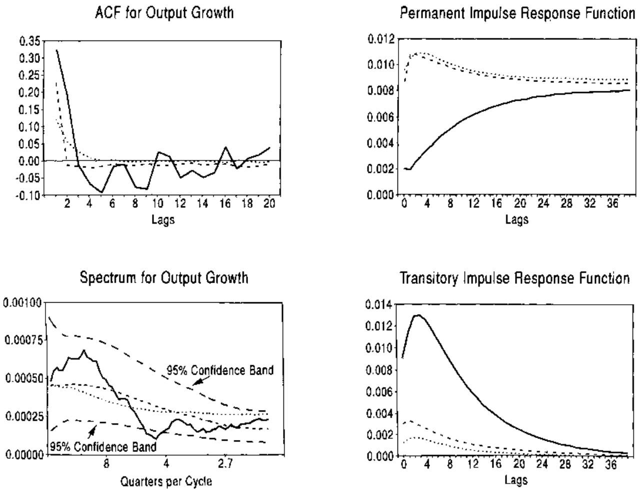


Figure 6. Employment Lags and Labor Adjustment Costs

Note: Solid lines show sample moments, dotted lines show moments generated by the cost-of-adjustment model, and dashed lines show moments generated by the Burnside et al. (1993) model.

Effects of gestation lags and adjustment costs (6/6)

Employment Lags and Labor Adjustment Costs			
		Q(irf)	
Model	Q(acf)	*y(p)*	*y(t)*
Burnside	6.7 (0.469)	31.5 (0.035)	72.7 (0.014)
Adjustment cost (0.36 , 2.2)	9.2 (0.326)	34.6 (0.031)	76.0 (0.012)
Sensitivity analysis			
(0.36/4 , 2.2)	12.9 (0.116)	37.0 (0.031)	193.7 (0.005)
(0.36/2 , 2.2)	9.8 (0.224)	39.2 (0.024)	123.1 (0.008)
(2*(0.36) , 2.2)	9.0 (0.339)	37.1 (0.024)	52.4 (0.021)
(2*(0.36) , 2.2)	10.1 (0.257)	48.7 (0.016)	54.8 (0.018)

- Problems:
  - both models overstate the short-term response of output to permanent shocks;
  - both models understate its response to transitory shocks.

Conclusions

- RBC models must rely heavily on exogenous factors to replicate both stylized facts.
- RBC models have weak internal propagation mechanisms. Then, they do not generate interesting dynamics via their internal structure.
- RBC theorist ought to devote attention to modeling internal sources of propagation.

Appendix 1: alternative RBC baseline models

MODEL	CHARACTERISTICS
King (1988)	Technology shocks are the only source of fluctuations
Hansen (1989)	Technology shocks are the only source of fluctuations
Greenwood (1988)	A) Technology shocks are the only source of fluctuations B) shocks only affect new capital goods, not existing C) Firms vary capacity utilization in response to variation in the user cost of capital
Braun (1994)	Includes distortionary taxes on Labor and Capital
Benhabib (1991)	Study a two-sector model in which goods are produced at home as well as in the market

Appendix: sample analog to the spectral representation theorem (1/2)

Let $y = \{y_1, y_2 .....y_T\}$. The value of y_t can be expressed as:

$$y_t = \bar{y} + \sum_{j=1}^{M} \left\{ \hat{\alpha}_j \cos\left[\omega_j(t-1)\right] + \hat{\delta}_j \sin\left[\omega_j(t-1)\right] \right\},$$

where $M = \frac{T-1}{2}$, $\omega_j = \frac{2\pi j}{T}$, $\bar{y}$ is the sample mean of y, and $\hat{\alpha}_j$ and $\hat{\delta}_j$ are given by:

$$\begin{aligned} \hat{\alpha}_j &= \frac{2}{T}\sum_{t=1}^{T} y_t \cos[\omega_j(t-1)], \\ \hat{\delta}_j &= \frac{2}{T}\sum_{t=1}^{T} y_t \sin[\omega_j(t-1)]. \end{aligned}$$

Appendix: sample analog to the spectral representation theorem (2/2)

Moreover the sample variance of y can be expressed as:

$$\frac{1}{T}\sum_{t=1}^{T}(y_t - \bar{y})^2 = \frac{1}{2}\sum_{j=1}^{M}(\hat{\alpha}_j^2 + \hat{\delta}_j^2).$$

The sample variance of y that can be attributed to cycles of frequency ω_j is given by $\frac{1}{2}(\hat{\alpha}_j^2 + \hat{\delta}_j^2)$.

Also we have that:

$$\frac{1}{2}(\hat{\alpha}_j^2 + \hat{\delta}_j^2) = \frac{4\pi}{T}\hat{s}_y(w_j),$$

where $\hat{s}_y(w_j)$ is the sample periodogram at frequency w_j.