Toulouse School of Economics – 2008-2009
M2 – Macroeconomics II — Franck Portier

Final Exam
Solution

I – Problem - The analytical RBC Model and the Real Wage - Employment Correlation *(40%)*

Let the model economy be populated with a representative household and a representative firm. The firm has a Cobb-Douglas technology:

$$Y_t = e^{z_t} K_t^{\gamma} N_t^{1-\gamma} \tag{1}$$

where K_t is capital, N_t labor input, and z_t a stochastic technological shift. All profits of the firm are distributed to the household. Capital evolves according to

$$K_{t+1} = I_t \tag{2}$$

where I_t is investment in period t.

The representative household works N_t and consumes C_t. Preferences are given by

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - e^{\chi_t} N_t\right] \tag{3}$$

where χ_t is a preference shock. Capital is accumulated by the household and rented to the firm. Let R_t denote the real rental rate of capital and W_t the real wage. The final good is chosen as the numéraire. It is assumed that χ and z are *i.i.d.* with respective variance σ_χ^2 and σ_z^2.

1 – Write the representative household problem and derive the FOCs.

The problem of the representative Hh writes

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - e^{\chi_t} N_t + \lambda_t (C_t + K_{t+1} - W_t N_t - R_t K_t)\right]$$

with K_0 given. The FOC are

$$\begin{aligned} \frac{1}{C_t} &= \lambda_t \\ e^{\chi_t} &= \lambda_t W_t \\ \lambda_t &= \beta E_t [R_{t+1} \lambda_{t+1}]. \end{aligned}$$

plus a transversality condition $\lim_{j \longrightarrow \infty} \beta^j \lambda_{t+j} K_{t+j+1}$.

2 – Write the representative firm problem and derive the FOCs.

The problem of the representative firm writes

$$\max \Pi_t = e^{z_t} K_t^{\gamma} N_t^{1-\gamma} - W_t N_t - R_t K_t.$$

The FOC are

$$\begin{aligned} \gamma \frac{Y_t}{K_t} &= R_t \\ (1-\gamma) \frac{Y_t}{N_t} &= W_t. \end{aligned}$$

3 – Define a competitive equilibrium of this economy

A competitive equilibrium of this economy is a vector of quantities (C_t, N_t, K_t, Y_t) and prices (R_t, W_t) for each period t such that

1. Quantities are maximizing utility and profit for given prices,
2. Prices are such that markets clear.

4 – Solve the model and show that the equilibrium process of output is $y_t = z_t + \gamma y_{t-1} - (1-\gamma)\chi_t$ (dropping constants and with the notation $x = \log X$)

Replacing λ and R by their expression in the Hh Euler equation, we obtain

$$\frac{1}{C_t} = \beta E_t \left[\gamma \frac{Y_{t+1}}{K_{t+1}} \frac{1}{C_{t+1}} \right].$$

Using $Y_{t+1} = C_{t+1} + I_{t+1}$ and $K_{t+1} = I_t$, we have

$$\frac{1}{C_t} = \beta E_t \left[\gamma \frac{C_{t+1} + It + 1}{I_t} \frac{1}{C_{t+1}} \right].$$

$$\Longleftrightarrow \frac{I_t}{C_t} = \beta\gamma + \beta\gamma E_t \left[\frac{I_t}{C_t} \right].$$

Iterating forward and using the transversality condition, we obtain

$$\frac{I_t}{C_t} = \frac{\beta\gamma}{1-\beta\gamma},$$

and therefore $C_t = (1-\beta\gamma)Y_t$ and $I_t = \beta\gamma Y_t$. From the Hh two first FOC, we obtain

$$N_t = \frac{1-\gamma}{1-\beta\gamma} e^{-\chi_t}.$$

Then replacing N_t and $K_t = I_{t-1}$ in the production function gives

$$Y_t = e^{z_t} \left(\beta\gamma Y_{t-1} \right)^{\gamma} \left(\frac{1-\gamma}{1-\beta\gamma} e^{-\chi_t} \right)^{1-\gamma}.$$

Taking logs and dropping constants gives

$$y_t = z_t + \gamma y_{t-1} - (1-\gamma)\chi_t.$$

5 – Derive the solution for the (log of the) real wage ω_t and for employment n_t (again dropping constants).

As already shown (in logs, dropping constants), $n_t = -\chi_t$. The real wage is given $\omega_t = y_t - n_t = z_t + \gamma y_{t-1} + \chi_t$.

6 – Compute and draw the IRF of y, ω and n to a technological and preference shock. Discuss.

If there is a shock on z such that $z_t = 0$ if $t \neq 0$ and $z_0 = 1$, then
⋆ $n_t = 0\ \forall t$
⋆ $\{\omega_t\}_{t=0}^{\infty} = \{y_t\}_{t=0}^{\infty} = \{1, \gamma, \gamma^2, \gamma^3, \ldots\}$.

If there is a shock on χ such that $\chi_t = 0$ if $t \neq 0$ and $\chi_0 = 1$, then
⋆ $\{n_t\}_{t=0}^{\infty} = \{-1, 0, 0, 0, \ldots\}$
⋆ $\{y_t\}_{t=0}^{\infty} = \{-(1-\gamma), -\gamma(1-\gamma), -\gamma^2(1-\gamma), -\gamma^3(1-\gamma), \ldots\}$
⋆ $\{\omega_t\}_{t=0}^{\infty}\{\gamma, -\gamma(1-\gamma), -\gamma^2(1-\gamma), -\gamma^3(1-\gamma), \ldots\}$.

See figures 1 and 2.

7 – Compute the correlation between ω_t and n_t. What do you know about the level of this correlation in the data. Discuss.

By definition,

$$cor(n_t, \omega_t) = \frac{cov(n_t, \omega_t)}{\sqrt{V(n_t)V(\omega_t)}}.$$

and we have
⋆ $cov(n_t, \omega_t) = cov(-\chi_t, z_t + \gamma y_{t-1} + \chi_t) = -\sigma_\chi^2$,
⋆ $V(n_t) = \sigma_\chi^2$,
⋆ $V(\omega_t) = \sigma_z^2 + \sigma_\chi^2$.
We therefore observe that $cor(n_t, \omega_t) < 0$. If $\sigma_z^2 = 0$, then $cor(n_t, \omega_t) = -1$. If $\sigma_\chi^2 = 0$, then $cor(n_t, \omega_t)$ is not defined as n is a constant.
Technology shocks imply a positive correlation between employment and the real wage in typical RBC models: the technology shock imply a large North-East shift of the labor demand schedule, and a small North-West shift of the labor supply one (because of the wealth effect). Therefore, both ω and n increase. In this particular model, the wealth effect is always exactly offsetting the substitution effect implied by the increase in ω. Therefore, n is constant and with only technology shock, the covariance between employment and the real wage is zero, and the correlation not defined.
Preference shocks imply that the household is likely to work less for the same real wage, which is an North-West shift of the labor supply schedule. As the labor demand schedule is not affected, the equilibrium moves along the labor demand curve, and the correlation between the real wage and employment is therefore negative. In this particular model, the larger is the relative variance of σ_χ^2/σ_z^2, the closer to one the correlation is.
In the data, we generally observe a very small correlation between employment and the real wage, meaning that both supply and demand shifts affect the economy. This particular model in not able to replicate the date, unless one assume very small (but non zero) variance of the preference shocks. In that case, the correlation goes to zero, but the variance of employment also goes to zero, which is not in line with the data (worked hours are about as volatile as output).

Figure 1: Impulse Responses to a technology shock z

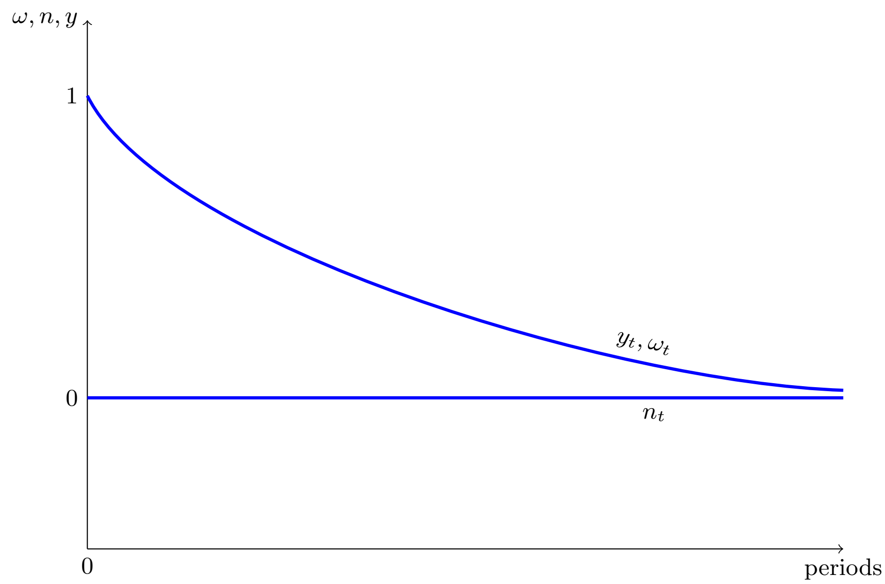
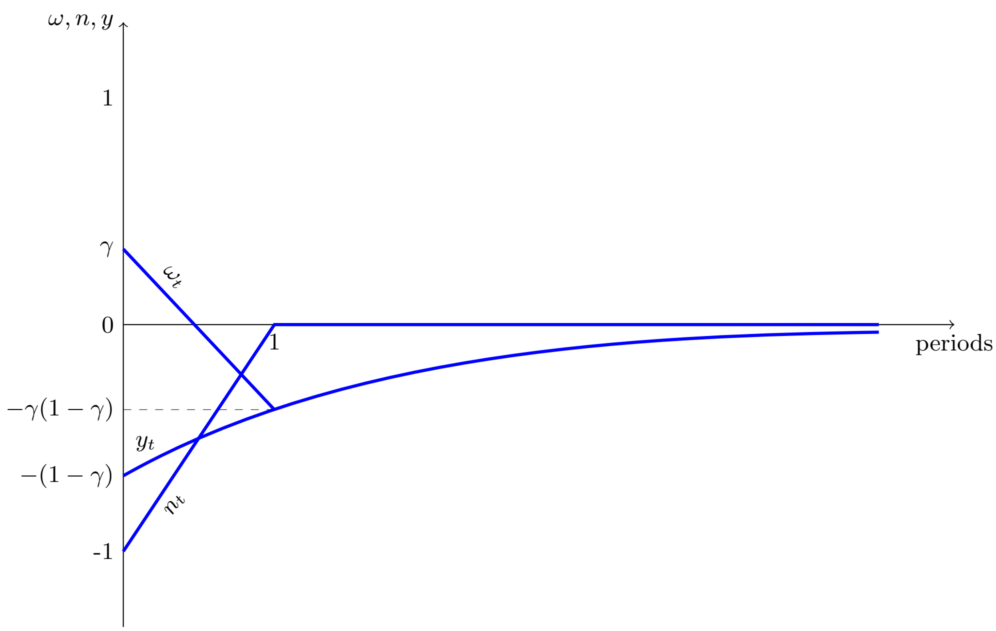


Figure 2: Impulse Responses to a preference shock χ



II – Questions *(30%)*

Please propose a structured answer to each question, with as much economic content as possible. Please define the main terms and use math if needed.

1 – Identification in structural VARs.

I. A VAR is a Vector Autoregressive Model. It is an easy to estimate time-series model. Let X be a $n \times 1$ vector of variables and ν a $n \times 1$ vector of residuals, on can estimate

$$X_t = \widetilde{A}(L)X_{t-1} + \nu_t,$$

with $\text{Var}(\nu) = \Omega$ and $C(0) = I$ by normalization. This VAR is a reduced form. It is non-structural in the sense that one cannot put names on the shocks ν. One would like to confront this estimation with a model in order to transform the ν into orthogonal and meaningful *shocks*. This is what we call identification in VARs.
We can write the Vector Moving Average representation of this process as

$$X_t = \sum_{j=0}^{\infty} C(j)\nu_{t-j}$$

.

II. Assume that the true model of the economy is

$$X_t = \widehat{A}(L)X_{t-1} + B\varepsilon_t$$

where ε are structural shocks (for example a monetary policy shock, a technology shock, a fiscal shock, a terms of trade shocks,...) and $\text{Var}(\varepsilon) = I_n$.
We can write the Vector Moving Average representation of this process as

$$X(t) = \sum_{j=0}^{\infty} A(j)\varepsilon_{t-j}$$

.

III. Comparing the two VMA representations of the same process, we obtain

$$\nu = A(0)\varepsilon \text{ and } A(j) = A(0)C(j) \text{ for } j > 0.$$

Estimation gives us C. We need to we know $A(0)$ to backup the structural shocks. To get $A(0)$, observe that if $\nu = A(0)\varepsilon$, then ν and $A(0)\varepsilon$ must have the same variance-covariance matrix. The one of ν is the Ω (estimated). The one of ε is I by assumption. Therefore, one has

$$V(A(0)\varepsilon) = V(\nu) \iff A(0)A(0)' = \Omega$$

.

IV. This last equality gives us $n \times (n+1)/2$ independent equations (because Ω and $A(0)A(0)'$ are symmetrical) for n^2 unknowns (the n^2 coefficients of $A(0)$). We need $n \times (n-1)/2$ extra equations (the identifying restrictions) to be able to obtain $A(0)$, and then ε. Those restrictions are not given by any mathematical or statistical theory, but are based on some "reasonable" properties of the economy.

V. Examples of identifying restrictions: "demand" shocks have no long run effect on real quantities, real variables do nor respond on impact to monetary policy shocks, ...

2 – Anticipated *versus* unanticipated economic policy.

I. Expectations matter for economic policy. Economic agents form expectations about the future and about the actions of the government.

II. Any model has (implicitly or explicitly) a theory on how agents of expectations. Expectations can be adaptive (a function of the past anticipation errors), static, rational.

III. Rational expectations correspond to a situation in which agents use in the best possible way the information that they have. Typically, one assumes that they know the model of the economy, the value of parameters, the process of shocks.

IV. In a most simple model, one would obtain solutions of the model of the type

$$X_t = \alpha E_{t-1} Z_t + \beta (Z_t - E_{t-1} Z_t),$$

where X is a vector of endogenous variables, Z a vector of economic policy variables and E_j the mathematical expectation conditional to the information of period j. The term $(Z_t - E_{t-1} Z_t)$ is the surprise in economic policy, and has *a priori* a different impact on the economy than the expected component $E_{t-1} Z_t$.

V. This disctinction if for example important in the debate on the slope of the aggregate supply curve, as illustrated by the "LUCAS supply curve"

$$y_t = \lambda y_{t-1} + \alpha (p_t - p_t^e),$$

where p^e stand for expected prices. Consider a simple AD-AS model where expectations are rational, the AS curve given by the "LUCAS one" and the AD equation

$$y_t = -\beta p_t + \gamma m_t.$$

In such a model, one can show (see the slides) that the solution writes

$$y_t = \frac{\alpha\gamma}{\alpha+\beta} \underbrace{(m_t - E_{t-1} m_t)}_{\text{surprise}} + \lambda y_{t-1}.$$

Anticipated monetary policy is inefficient $\rightsquigarrow$ the AS curve is vertical on average. Only monetary surprises are efficient $\rightsquigarrow$ non systematic effect of monetary policy. A feedback rule of the type $m_t = \zeta(\overline{y} - y_{t-1})$ is inefficient ($\overline{y} = 0$ is the non stochastic equilibrium level of output, that we assume here to be a target for the Central Bank).

III – Discussion – About Gali, Gertler and López-Salido 2001 EER Paper (*European inflation dynamics*) (30%)

Below is the abstract of a paper published in 2001 in the European Economic Review by Jordi Galí, Mark Gertler and David López-Salido on European inflation dynamics.

Abstract

We provide evidence on the fit of the New Phillips Curve (NPC) for the Euro area over the period 1970–1998, and use it as a tool to compare the characteristics of European inflation dynamics with those observed in the U.S. We also analyze the factors underlying inflation inertia by examining the cyclical behavior of marginal costs, as well as that of its two main components, namely, labor productivity and real wages. Some of the findings can be summarized as follows: (a) the NPC fits Euro area data very well, possibly better than U.S. data, (b) the degree of price stickiness implied by the estimates is substantial, but in line with survey evidence and U.S. estimates, (c) inflation dynamics in the Euro area appear to have a stronger forward-looking component (i.e., less inertia) than in the U.S., (d) labor market frictions, as manifested in the behavior of the wage markup, appear to have played a key role in shaping the behavior of marginal costs and, consequently, inflation in Europe.

JEL classification: E31

Keywords: Inflation; Phillips curve; EMU

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1 – Relate Extract 1 (displayed on the next page) to what you know about the role of the Phillips Curve in traditional AD-AS models. Why is the case where the coefficients on lagged inflation sum to one particularly relevant?

I. The Phillips curve (PC) is originally an empirical (negative) relation between inflation and unemployment.
II. It served as the "missing" equation in the AD/AS model, as it was providing an equation for price adjustment.
III. In the 1970's, the PC lost its empirical support, and Friedman and Phelps, followed by Lucas showed how expectations modeling undermined its theoretical support.
IV. The PC was shown to be theoretically vertical not only in the long run, but also possibly in the short run if expectations were rational.
V. Note that a vertical PC corresponds to a situation in which output does not depend on inflation in the long run. If we consider the equation of the text:

$$\pi_t = \sum_{i=1}^{h} \varphi_i \pi_{t-i} + \delta \widehat{y}_{t-1} + \varepsilon_t,$$

the (deterministic) long run corresponds to $\pi_t = \pi \quad \forall t$. In the long run,

$$\widehat{y} = \frac{1}{\delta} \left(\left(1 - \sum_{i=1}^{h} \varphi_i \right) \pi \right).$$

If $\sum_{i=1}^{h} \varphi_i = 1$, then $\widehat{y} = 0$: the output gap is zero in the long run, it does not depend on inflation, the PC is vertical.

2 – Interpret equation (2) of Extract 2 in relation with the course on the construction of the New-Keynesian Phillips Curve.

I. The New-Keynesian model (NKM) considers that firms (monopolistic competitors) randomly reset their prices (Calvo model.
II. As firms are monopolistic, they set the price as a markup over marginal cost.
III. Because they don't set their price every period, they decide of a markup over current and expected marginal costs, whichh explains that the NK PC is forward-looking.

3 – What are the different ways of computing an output gap (to be defined)? What is the New-Keynesian monetary model suggesting?

I. Broadly speaking, the output gap is the difference between actual GDP and its trend.
II. As seen in the course, they are many ways of decomposing a non stationary time series into a trend and a cycle: by computing potential output, by removing a linear trend, by taking first-differences, by taking an HODRICK-PRESCOTT filter,...
III. The NK model suggest a structural relation of inflation with the marginal cost and not the output gap. According to the authors, the real unit labor cost is a good proxy for the marginal cost.

4 − What is the meaning of parameter λ in equation (10) of Extract 3. Explain the effect of parameters θ, α and ε on the value of λ. *[The production function of a firm j is $Y_j = AN_j^{1-\alpha}$ and the demand addressed to firm j is $Y_j = (P_j/P)^{-\varepsilon}\, Y$, where Y and P are aggregate quantity and price indexes].*

I. λ relates inflation to the (average, discounted) marginal cost.
II. If prices are very sticky (θ high), inflation moves very little (λ low).
II. When α is large, production is almost linear in labor, so that the marginal productivity of labor is almost constant. As marginal productivity is related to the real wage, the real wage is almost constant, so that being not exactly at the optimal production scale does not matter a lot for the firm. Therefore, the firm needs not to manipulate prices: inflation is not very sensitive to the marginal cost (λ low). The same argument applies for the elasticity of demand.

5 − Discuss the results of Extract 4.

I. Results show that only the specification suggested by the model (using the real unit labor cost) gives good results.
II. By good, we mean that the coefficients are precisely estimated and have the "correct" sign.
III. Note that the coefficient on lagged inflation is very close to one, not significantly different from one, so that one cannot reject that the PC is vertical in the long run.

2.1. The traditional Phillips curve

The traditional Phillips curve relates inflation to some cyclical indicator plus lagged values of inflation. For example, let π_t denote inflation and $\hat{y}_t$ the log deviation of real GDP from its long run trend. A common specification of the traditional Phillips curve is

$$\pi_t = \sum_{i=1}^{h} \varphi_i \pi_{t-i} + \delta \hat{y}_{t-1} + \varepsilon_t, \tag{1}$$

where ε_t is a random disturbance. Often the restriction is imposed that the sum of the weights on lagged inflation is unity, so that the model implies no long run trade-off between output and inflation. Sometimes the equation includes additional lags of detrended output. Alternative specifications may use different cyclical indicators (e.g., the unemployment rate, capacity utilization, etc.)

Despite considerable criticism, however, the traditional Phillips curve does a reasonable job of characterizing post war inflation in the U.S. For example, Rudebusch and Svensson (1999, henceforth RS) show that a variant of Eq. (1) with four lags of inflation fits well quarterly U.S. data over the period 1960–1999[5]. The output term enters significantly with a positive sign and the sum of the coefficients on lagged inflation does not differ significantly from unity.

Here we show that the traditional Phillips curve similarly appears to provide a reasonable description of inflation in the Euro area, over the available sample. To measure inflation we use the log difference of the GDP deflator. The output term is the log of real GDP, detrended with a fitted quadratic function of time. Estimates of the RS specification of Eq. (1) for quarterly Euro area data over the sample 1970:I–1998:II yield

$$\pi_t = \underset{(0.087)}{0.520}\ \pi_{t-1} + \underset{(0.073)}{0.233}\ \pi_{t-2} - \underset{(0.084)}{0.070}\ \pi_{t-3} + \underset{(0.086)}{0.256}\ \pi_{t-4} + \underset{(0.016)}{0.051}\ \hat{y}_{t-1} + \varepsilon_t.$$

For comparison, estimates of the model for U.S. data over the same sample yield

$$\pi_t = \underset{(0.041)}{0.602}\ \pi_{t-1} + \underset{(0.153)}{0.041}\ \pi_{t-2} + \underset{(0.119)}{0.152}\ \pi_{t-3} + \underset{(0.055)}{0.155}\ \pi_{t-4} + \underset{(0.014)}{0.048}\ \hat{y}_{t-1} + \varepsilon_t.$$

Not only does the RS specification appear to work well for the Euro area, the estimated coefficients are quite similar to those obtained for U.S. data.

2.2. The new Phillips curve

The new Phillips curve is based on staggered nominal price setting, in the spirit of Taylor's (1980) seminal work. A key difference is that price setting behavior is the product of optimization by monopolistically competitive firms subject to constraints on the frequency of price adjustment. A popular example is based on Calvo's model (1983) of staggered price setting, which has the virtue of parsimony. Here we outline the key aspects, and defer some of the details relevant for an explicit derivation of an estimable relation to Section 3.1 below.

The basic building block is the following equation that relates inflation π_t to anticipated future inflation and real marginal cost:

$$\pi_t = \beta \mathrm{E}_t\{\pi_{t+1}\} + \lambda \widehat{mc}_t, \tag{2}$$

where $\widehat{mc}_t$ is average real marginal cost, in percent deviation from its steady-state level, β is a subjective discount factor, and λ is a slope coefficient that depends on the primitive parameters of the model, particularly the parameter that governs the degree of price rigidity. Eq. (2) is a log-linear approximation of a relation obtained from aggregating across the pricing decisions of individual firms.[7] This relation is what we referred to in the introduction as the 'primitive formulation' of the new Phillips curve; i.e., it is the formulation that arises directly as a consequence of the frictions in the price adjustment process that are the key aspect of the theory.

What is most often seen in the literature, however, is the 'standard formulation' of the new Phillips curve that instead relates inflation to an *output gap* variable. Under certain restrictions on technology and labor market structure (see, e.g., Rotemberg and Woodford, 1997), within a local neighborhood of the steady-state real marginal costs are proportionately related to the output gap as follows:

$$\widehat{mc}_t = \delta(y_t - y_t^*), \tag{3}$$

where y_t and y_t^* are the logarithms of real output and the natural level of real output, respectively. Combining (2) with (3) then yields the standard output gap-based formulation of the new Phillips curve:

$$\pi_t = \beta \mathrm{E}_t\{\pi_{t+1}\} + \kappa(y_t - y_t^*), \tag{4}$$

where $\kappa = \lambda\delta$.

It is Eq. (4) that has been the subject of considerable controversy. As with the traditional Phillips curve, inflation varies positively with the output gap. In contrast to the traditional Phillips curve, however, inflation is an entirely forward looking phenomenon. Iterating Eq. (4) forward yields

$$\pi_t = \kappa \sum_{k=0}^{\infty} \beta^k \mathrm{E}_t\{(y_{t+k} - y^*_{t+k})\}. \tag{5}$$

A striking implication is the absence of a tradeoff between inflation and output; to the extent a central bank can commit to stabilizing the output gap $(y_{t+k} - y^*_{t+k})$, it can achieve price stability. However, as emphasized by Fuhrer and Moore (1995), GG and others, Eq. (5) is at odds with the data. It suggests that inflation should anticipate movements in the output gap.[8] Yet, as the estimates of the traditional Phillips curve suggest, the output gap (measured by detrended output) tends to lead inflation.[9] While this result is widely known to hold for U.S. data, our Phillips curve estimates in the previous section suggest that it applies equally well to the Euro area. Overall, the output-gap based formulation of the new Phillips curve cannot account for the persistence of inflation either for the U.S. or for the Euro area.

As we noted in the introduction, however, Sbordone (1999) and GG find that the central aspect of the theory, the relation between inflation and real marginal cost given by Eq. (2) is roughly consistent with the data (see footnote 4). These results suggest that it is Eq. (3), the hypothesized link between real marginal cost and the output gap, that is at variance with the data. GG present some direct evidence for U.S. data to show that this is indeed the case. Real marginal cost tends to respond sluggishly and with a lag to movements in the output gap, much as inflation does. There are two possible explanations for this finding. One is that conventional measures of the output gap may be poor. To the extent that there are significant real shocks to the economy (e.g., shifts in technology growth, fiscal shocks, etc.), using detrended output as a proxy for y^*_t may not be appropriate. Whether this factor alone could account for the observed inertia in real marginal cost relative to detrended output is an open question, however.

A second, and perhaps more likely possibility, is that even if the output gap is correctly measured, it may not be the case that real marginal cost moves proportionately, as assumed. In particular, as we discuss in Section 5, with frictions in the labor market, either, in the form of real or nominal wage rigidities, Eq. (3) is no longer valid. These labor market rigidities, further, can in principle offer a rationale for the inertial behavior of real marginal cost.[10] Indeed, in Section 5 we provide evidence that labor market frictions were an important factor in the dynamics of marginal cost for both the Euro area and the U.S., though with some important differences across the two regions.

3. A marginal cost-based Phillips curve

In this section we derive a structural relation between inflation and average real marginal cost across firms that we estimate in the subsequent section. As in GG, we first present a baseline model. We then derive a hybrid model that allows for a fraction of firms to set prices using a backward looking rule of thumb. Here the idea is to test the baseline model explicitly against the alternative that arbitrary lags of inflation are required to explain inflation, as in the traditional Phillips curve analysis.

One difference from GG is that we relax the assumption that firms face identical constant marginal costs (which greatly simplifies aggregation), and instead allow for increasing real marginal cost, following Woodford (1996) and Sbordone (1999). We choose this path because allowing marginal cost to vary across firms produces more plausible estimates of the degree of price rigidity in the Euro area. Our baseline model, accordingly, is exactly the theoretical framework in Sbordone (1999). Our hybrid model is a generalization that extends GG to allow for increasing marginal cost. The appendix provides a detailed derivation.

We obtain the primitive formulation of the new Phillips curve that relates inflation to real marginal cost by combining Eqs. (6), (7), and (9):

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \lambda \widehat{mc}_t \tag{10}$$

with

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)(1-\alpha)}{\theta[1+\alpha(\varepsilon-1)]}. \tag{11}$$

Note that the slope coefficient λ depends on the primitive parameters of the model. In particular, λ is decreasing in the degree of price rigidity, as measured by θ, the fraction of firms that keep their prices constant. A smaller fraction of firms adjusting prices implies that inflation will be less sensitive to movements in marginal cost. Second, λ is also decreasing in the curvature of the production function, as measured by α, and in the elasticity of demand ε. The larger α and ε, the more sensitive is the marginal cost of an individual firm to deviations of its price from the average price level: everything else equal, a smaller adjustment in price is desirable in order to offset expected movements in average marginal costs.

Finally, we observe that Eq. (10) can be expressed completely in terms of observables, since (8) implies that average real marginal costs correspond to real unit labor costs (or, equivalently, to the labor income share).[12] In the end, accordingly, the model suggests that inflation should equal a discounted stream of expected future real unit labor costs.

4. Evidence

We next present estimates of both the baseline model (Eq. (10)) and the hybrid model (Eq. (12)) for the Euro area. For comparison, we also present results for the U.S. over the same sample period.

All data are quarterly time series over the period 1970:I–1998:II. To measure inflation we use the GDP deflator. Fig. 1 plots that variable, as well as detrended GDP. Our measure of average real marginal cost is the log of real unit labor costs, consistent with the theory presented on Section 3.1.[14] Accordingly, we use the log deviation of real unit labor costs from its mean as a measure of $\widehat{mc}_t$.

The estimated inflation equation for the Euro area is given by

$$\pi_t = \underset{(0.040)}{0.914}\ E_t\{\pi_{t+1}\} + \underset{(0.041)}{0.088}\ \widehat{mc}_t, \tag{13}$$

where standard errors are shown in parentheses. The corresponding equation for the U.S. is

$$\pi_t = \underset{(0.029)}{0.924}\ E_t\{\pi_{t+1}\} + \underset{(0.118)}{0.250}\ \widehat{mc}_t. \tag{14}$$

By way of contrast, when we estimate the model using detrended log GDP (as a proxy for the output gap, following other authors), the slope coefficient becomes the wrong sign:

$$\pi_t = \underset{(0.018)}{0.990}\ E_t\{\pi_{t+1}\} - \underset{(0.007)}{0.003}\ \hat{y}_t \tag{15}$$

and the corresponding equation for the U.S. yields the same conclusion:

$$\pi_t = \underset{(0.026)}{1.012}\ E_t\{\pi_{t+1}\} - \underset{(0.006)}{0.021}\ \hat{y}_t. \tag{16}$$