Toulouse School of Economics – 2008-2009
M2 – Macroeconomics II — Franck Portier

Final Exam

I – Problem - The analytical RBC Model and the Real Wage - Employment Correlation *(40%)*

Let the model economy be populated with a representative household and a representative firm. The firm has a Cobb-Douglas technology:

$$Y_t = e^{z_t} K_t^{\gamma} N_t^{1-\gamma} \tag{1}$$

where K_t is capital, N_t labor input, and z_t a stochastic technological shift. All profits of the firm are distributed to the household. Capital evolves according to

$$K_{t+1} = I_t \tag{2}$$

where I_t is investment in period t.

The representative household works N_t and consumes C_t. Preferences are given by

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - e^{\chi_t} N_t\right] \tag{3}$$

where χ_t is a preference shock. Capital is accumulated by the household and rented to the firm. Let R_t denote the real rental rate of capital and W_t the real wage. The final good is chosen as the numéraire. It is assumed that χ and z are *i.i.d.* with respective variance σ_χ^2 and σ_z^2.

1 – Write the representative household problem and derive the FOCs.

2 – Write the representative firm problem and derive the FOCs.

3 – Define a competitive equilibrium of this economy

4 – Solve the model and show that the equilibrium process of output is $y_t = z_t + \gamma y_{t-1} - (1-\gamma)\chi_t$ (dropping constants and with the notation $x = \log X$)

5 – Derive the solution for the (log of the) real wage ω_t and for employment n_t (again dropping constants).

6 – Compute and draw the IRF of y, ω and n to a technological and preference shock. Discuss.

7 – Compute the correlation between ω_t and n_t. What do you know about the level of this correlation in the data. Discuss.

II – Questions *(30%)*

Please propose a structured answer to each question, with as much economic content as possible. Please define the main terms and use math if needed.

1 – Identification in structural VARs.

2 – Anticipated *versus* unanticipated economic policy.

III – Discussion – About Gali, Gertler and López-Salido 2001 EER Paper (*European inflation dynamics*) (*30%*)

Below is the abstract of a paper published in 2001 in the European Economic Review by Jordi Galí, Mark Gertler and David López-Salido on European inflation dynamics.

Abstract

We provide evidence on the fit of the New Phillips Curve (NPC) for the Euro area over the period 1970–1998, and use it as a tool to compare the characteristics of European inflation dynamics with those observed in the U.S. We also analyze the factors underlying inflation inertia by examining the cyclical behavior of marginal costs, as well as that of its two main components, namely, labor productivity and real wages. Some of the findings can be summarized as follows: (a) the NPC fits Euro area data very well, possibly better than U.S. data, (b) the degree of price stickiness implied by the estimates is substantial, but in line with survey evidence and U.S. estimates, (c) inflation dynamics in the Euro area appear to have a stronger forward-looking component (i.e., less inertia) than in the U.S., (d) labor market frictions, as manifested in the behavior of the wage markup, appear to have played a key role in shaping the behavior of marginal costs and, consequently, inflation in Europe.

JEL classification: E31

Keywords: Inflation; Phillips curve; EMU

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1 – Relate Extract 1 (displayed on the next page) to what you know about the role of the Phillips Curve in traditional AD-AS models. Why is the case where the coefficients on lagged inflation sum to one particularly relevant?

2 – Interpret equation (2) of Extract 2 in relation with the course on the construction of the New-Keynesian Phillips Curve.

3 – What are the different ways of computing an output gap (to be defined)? What is the New-Keynesian monetary model suggesting?

4 – What is the meaning of parameter λ in equation (10) of Extract 3. Explain the effect of parameters θ, α and ε on the value of λ. *[The production function of a firm j is $Y_j = AN_j^{1-\alpha}$ and the demand addressed to firm j is $Y_j = (P_j/P)^{-\varepsilon}\, Y$, where Y and P are aggregate quantity and price indexes].*

5 – Discuss the results of Extract 4.

2.1. The traditional Phillips curve

The traditional Phillips curve relates inflation to some cyclical indicator plus lagged values of inflation. For example, let π_t denote inflation and $\hat{y}_t$ the log deviation of real GDP from its long run trend. A common specification of the traditional Phillips curve is

$$\pi_t = \sum_{i=1}^{h} \varphi_i \pi_{t-i} + \delta \hat{y}_{t-1} + \varepsilon_t, \tag{1}$$

where ε_t is a random disturbance. Often the restriction is imposed that the sum of the weights on lagged inflation is unity, so that the model implies no long run trade-off between output and inflation. Sometimes the equation includes additional lags of detrended output. Alternative specifications may use different cyclical indicators (e.g., the unemployment rate, capacity utilization, etc.)

Despite considerable criticism, however, the traditional Phillips curve does a reasonable job of characterizing post war inflation in the U.S. For example, Rudebusch and Svensson (1999, henceforth RS) show that a variant of Eq. (1) with four lags of inflation fits well quarterly U.S. data over the period 1960–1999[5]. The output term enters significantly with a positive sign and the sum of the coefficients on lagged inflation does not differ significantly from unity.

Here we show that the traditional Phillips curve similarly appears to provide a reasonable description of inflation in the Euro area, over the available sample. To measure inflation we use the log difference of the GDP deflator. The output term is the log of real GDP, detrended with a fitted quadratic function of time. Estimates of the RS specification of Eq. (1) for quarterly Euro area data over the sample 1970:I–1998:II yield

$$\pi_t = \underset{(0.087)}{0.520}\ \pi_{t-1} + \underset{(0.073)}{0.233}\ \pi_{t-2} - \underset{(0.084)}{0.070}\ \pi_{t-3} + \underset{(0.086)}{0.256}\ \pi_{t-4} + \underset{(0.016)}{0.051}\ \hat{y}_{t-1} + \varepsilon_t.$$

For comparison, estimates of the model for U.S. data over the same sample yield

$$\pi_t = \underset{(0.041)}{0.602}\ \pi_{t-1} + \underset{(0.153)}{0.041}\ \pi_{t-2} + \underset{(0.119)}{0.152}\ \pi_{t-3} + \underset{(0.055)}{0.155}\ \pi_{t-4} + \underset{(0.014)}{0.048}\ \hat{y}_{t-1} + \varepsilon_t.$$

Not only does the RS specification appear to work well for the Euro area, the estimated coefficients are quite similar to those obtained for U.S. data.

2.2. The new Phillips curve

The new Phillips curve is based on staggered nominal price setting, in the spirit of Taylor's (1980) seminal work. A key difference is that price setting behavior is the product of optimization by monopolistically competitive firms subject to constraints on the frequency of price adjustment. A popular example is based on Calvo's model (1983) of staggered price setting, which has the virtue of parsimony. Here we outline the key aspects, and defer some of the details relevant for an explicit derivation of an estimable relation to Section 3.1 below.

The basic building block is the following equation that relates inflation π_t to anticipated future inflation and real marginal cost:

$$\pi_t = \beta \mathrm{E}_t\{\pi_{t+1}\} + \lambda \widehat{mc}_t, \tag{2}$$

where $\widehat{mc}_t$ is average real marginal cost, in percent deviation from its steady-state level, β is a subjective discount factor, and λ is a slope coefficient that depends on the primitive parameters of the model, particularly the parameter that governs the degree of price rigidity. Eq. (2) is a log-linear approximation of a relation obtained from aggregating across the pricing decisions of individual firms.[7] This relation is what we referred to in the introduction as the 'primitive formulation' of the new Phillips curve; i.e., it is the formulation that arises directly as a consequence of the frictions in the price adjustment process that are the key aspect of the theory.

What is most often seen in the literature, however, is the 'standard formulation' of the new Phillips curve that instead relates inflation to an *output gap* variable. Under certain restrictions on technology and labor market structure (see, e.g., Rotemberg and Woodford, 1997), within a local neighborhood of the steady-state real marginal costs are proportionately related to the output gap as follows:

$$\widehat{mc}_t = \delta(y_t - y_t^*), \tag{3}$$

where y_t and y_t^* are the logarithms of real output and the natural level of real output, respectively. Combining (2) with (3) then yields the standard output gap-based formulation of the new Phillips curve:

$$\pi_t = \beta \mathrm{E}_t\{\pi_{t+1}\} + \kappa(y_t - y_t^*), \tag{4}$$

where $\kappa = \lambda\delta$.

It is Eq. (4) that has been the subject of considerable controversy. As with the traditional Phillips curve, inflation varies positively with the output gap. In contrast to the traditional Phillips curve, however, inflation is an entirely forward looking phenomenon. Iterating Eq. (4) forward yields

$$\pi_t = \kappa \sum_{k=0}^{\infty} \beta^k \mathrm{E}_t\{(y_{t+k} - y^*_{t+k})\}. \tag{5}$$

A striking implication is the absence of a tradeoff between inflation and output; to the extent a central bank can commit to stabilizing the output gap $(y_{t+k} - y^*_{t+k})$, it can achieve price stability. However, as emphasized by Fuhrer and Moore (1995), GG and others, Eq. (5) is at odds with the data. It suggests that inflation should anticipate movements in the output gap.[8] Yet, as the estimates of the traditional Phillips curve suggest, the output gap (measured by detrended output) tends to lead inflation.[9] While this result is widely known to hold for U.S. data, our Phillips curve estimates in the previous section suggest that it applies equally well to the Euro area. Overall, the output-gap based formulation of the new Phillips curve cannot account for the persistence of inflation either for the U.S. or for the Euro area.

As we noted in the introduction, however, Sbordone (1999) and GG find that the central aspect of the theory, the relation between inflation and real marginal cost given by Eq. (2) is roughly consistent with the data (see footnote 4). These results suggest that it is Eq. (3), the hypothesized link between real marginal cost and the output gap, that is at variance with the data. GG present some direct evidence for U.S. data to show that this is indeed the case. Real marginal cost tends to respond sluggishly and with a lag to movements in the output gap, much as inflation does. There are two possible explanations for this finding. One is that conventional measures of the output gap may be poor. To the extent that there are significant real shocks to the economy (e.g., shifts in technology growth, fiscal shocks, etc.), using detrended output as a proxy for y^*_t may not be appropriate. Whether this factor alone could account for the observed inertia in real marginal cost relative to detrended output is an open question, however.

A second, and perhaps more likely possibility, is that even if the output gap is correctly measured, it may not be the case that real marginal cost moves proportionately, as assumed. In particular, as we discuss in Section 5, with frictions in the labor market, either, in the form of real or nominal wage rigidities, Eq. (3) is no longer valid. These labor market rigidities, further, can in principle offer a rationale for the inertial behavior of real marginal cost.[10] Indeed, in Section 5 we provide evidence that labor market frictions were an important factor in the dynamics of marginal cost for both the Euro area and the U.S., though with some important differences across the two regions.

3. A marginal cost-based Phillips curve

In this section we derive a structural relation between inflation and average real marginal cost across firms that we estimate in the subsequent section. As in GG, we first present a baseline model. We then derive a hybrid model that allows for a fraction of firms to set prices using a backward looking rule of thumb. Here the idea is to test the baseline model explicitly against the alternative that arbitrary lags of inflation are required to explain inflation, as in the traditional Phillips curve analysis.

One difference from GG is that we relax the assumption that firms face identical constant marginal costs (which greatly simplifies aggregation), and instead allow for increasing real marginal cost, following Woodford (1996) and Sbordone (1999). We choose this path because allowing marginal cost to vary across firms produces more plausible estimates of the degree of price rigidity in the Euro area. Our baseline model, accordingly, is exactly the theoretical framework in Sbordone (1999). Our hybrid model is a generalization that extends GG to allow for increasing marginal cost. The appendix provides a detailed derivation.

We obtain the primitive formulation of the new Phillips curve that relates inflation to real marginal cost by combining Eqs. (6), (7), and (9):

$$\pi_t = \beta \mathrm{E}_t\{\pi_{t+1}\} + \lambda \widehat{mc}_t \tag{10}$$

with

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)(1-\alpha)}{\theta[1+\alpha(\varepsilon-1)]}. \tag{11}$$

Note that the slope coefficient λ depends on the primitive parameters of the model. In particular, λ is decreasing in the degree of price rigidity, as measured by θ, the fraction of firms that keep their prices constant. A smaller fraction of firms adjusting prices implies that inflation will be less sensitive to movements in marginal cost. Second, λ is also decreasing in the curvature of the production function, as measured by α, and in the elasticity of demand ε. The larger α and ε, the more sensitive is the marginal cost of an individual firm to deviations of its price from the average price level: everything else equal, a smaller adjustment in price is desirable in order to offset expected movements in average marginal costs.

Finally, we observe that Eq. (10) can be expressed completely in terms of observables, since (8) implies that average real marginal costs correspond to real unit labor costs (or, equivalently, to the labor income share).[12] In the end, accordingly, the model suggests that inflation should equal a discounted stream of expected future real unit labor costs.

4. Evidence

We next present estimates of both the baseline model (Eq. (10)) and the hybrid model (Eq. (12)) for the Euro area. For comparison, we also present results for the U.S. over the same sample period.

All data are quarterly time series over the period 1970:I–1998:II. To measure inflation we use the GDP deflator. Fig. 1 plots that variable, as well as detrended GDP. Our measure of average real marginal cost is the log of real unit labor costs, consistent with the theory presented on Section 3.1.[14] Accordingly, we use the log deviation of real unit labor costs from its mean as a measure of $\widehat{mc}_t$.

The estimated inflation equation for the Euro area is given by

$$\pi_t = \underset{(0.040)}{0.914}\ \mathrm{E}_t\{\pi_{t+1}\} + \underset{(0.041)}{0.088}\ \widehat{mc}_t, \tag{13}$$

where standard errors are shown in parentheses. The corresponding equation for the U.S. is

$$\pi_t = \underset{(0.029)}{0.924}\ \mathrm{E}_t\{\pi_{t+1}\} + \underset{(0.118)}{0.250}\ \widehat{mc}_t. \tag{14}$$

By way of contrast, when we estimate the model using detrended log GDP (as a proxy for the output gap, following other authors), the slope coefficient becomes the wrong sign:

$$\pi_t = \underset{(0.018)}{0.990}\ \mathrm{E}_t\{\pi_{t+1}\} - \underset{(0.007)}{0.003}\ \hat{y}_t \tag{15}$$

and the corresponding equation for the U.S. yields the same conclusion:

$$\pi_t = \underset{(0.026)}{1.012}\ \mathrm{E}_t\{\pi_{t+1}\} - \underset{(0.006)}{0.021}\ \hat{y}_t. \tag{16}$$