Toulouse School of Economics – 2007-2008
M2 – Macroeconomics II — Fabrice Collard & Franck Portier

# Final Exam

## I – Problem - Rational Expectations and Staggered Prices *(50 points)*

We consider here a model inspired from the works of Stan Fisher (1977) and John Taylor (1978). Consider an economy with a continuum of individuals indexed by $i$, and uniformly distributed over the interval $[0,1]$. The advantage of considering a measure 1 of individuals is that, if $x_i$ denotes a individual variable, the aggregate counterpart will be $x=\int_0^1 x_i \mathrm{d}i$. At a symmetric allocation, we will have $x_i = x \ \ \forall i$.

**Price setting with imperfect competition and flexible prices:** Each individual is the only producer of good $i$, that is produced in quantity $Q_i$ according to the technology $Q_i=\widetilde{L}_i$, where $\widetilde{L}_i$ is the amount of labor hired by agent $i$. Agent $i$ supplies $L_i$ units of labor on a single labor market, and may work in any firm. $W$ is the nominal wage in the economy. Utility of agent $i$ is given by

$$U_i = C_i - \frac{L_i^\gamma}{\gamma}, \qquad \gamma>1. \tag{1}$$

where $C_i$ is agent $i$ consumption. It is a basket of all the goods produced in the economy, with a price $P=\int_0^1 P_i \mathrm{d}i$. Agent $i$ nominal income $I_i$ is the sum profits $P_iQ_i - W\widetilde{L}_i$ and labor income $WL_i$. It is assumed that good $i$ demand is given by

$$Q_i = Y\left(\frac{P_i}{P}\right)^{-\eta}, \qquad \eta>1 \tag{2}$$

where $Y=\int_0^1 Q_i \mathrm{d}i$ is aggregate production and also aggregate real income. Finally, aggregate demand is given by

$$PY = M \tag{3}$$

where $M$ is the exogenous money supply. Money is the numéraire.

**1 –** Discuss, interpret, give foundations to equation (3).

**2 –** Show that the utility maximization problem of agent $i$ reduces to

$$\max_{P_i,L_i} \frac{(P_i-W)Y(P_i/P)^{-\eta}+WL_i}{P} - \frac{L_i^\gamma}{\gamma}$$

**3 –** Derive the first order conditions of the utility maximization problem. Manipulate those equations to obtain *(i)* an equation that expresses the relative price $P_i/P$ as a markup over marginal cost and *(ii)* a labor supply equation that expresses $L_i$ as a function of the real wage.

**4 –** Solve for the symmetric equilibrium. [*Hint: in equilibrium, $x_i = x = \int_0^1 x_i di$ for any variable $x$ of the model, and the good market equilibrium conditions writes $C=Q=Y$.*]. Give the equilibrium value of $Y$ and $P$. Compute the money multiplier and discuss. Discuss of the effect on $\eta$ on output level.

**5 –** Define $p_i^\star$ as the log of the optimal (flexible) price for agent $i$. Show that

$$p_i^\star = c + (1-\phi)p + \phi m$$

where $p=\log(P)$, $m=\log(M)$. Give the value of $c$ and $\phi$. This price will be referred to as the target price when prices will be predetermined or fixed.

**The model with predetermined prices and rational expectations (the Fisher Model):** We now assume that half of the agents sets their price in odd periods, half in even ones. When an agent sets prices in period, $t$, she does set the next period $(t+1)$ price and the price of the period after $(t+2)$, at the expected target levels of period $t+1$ and $t+2$. Prices needs not to be the same in periods $t+1$ and $t+2$, but they are predetermined. In any period, half of prices are ones set in the previous period and half are ones set two periods ago. Thus, the average (log) price is

$$p_t = \frac{1}{2}(p_t^1 + p_t^2)$$

where $p_t^1$denotes the price set for $t$ by individuals who set their prices in $t-1$ and $p_t^2$ the price set for $t$ by individuals setting prices in $t-2$. $p_t^1$ equals the expectation as of period $t-1$ of $p_{it}^\star$ ($p_t^1 = E_{t-1}\ p_{it}^\star$) and $p_t^2$ equals the expectation as of $t-2$ of $p_{it}^\star$ ($p_t^2 = E_{t-2}\ p_{it}^\star$). In the following, we consider for simplicity the model without the constant $c$:

$$p_{it}^\star = (1-\phi)p_t + \phi m_t.$$

Note also that we have added time subscripts.

**6 –** Express $p_t^1$ as a function of $E_{t-1}\ m_t$, $p_t^1$ and $p_t^2$. Express $p_t^2$ as a function of $E_{t-2}\ m_t$, $E_{t-2}\ p_t^1$ and $p_t^2$.

**7 –** Solve for $p_t^1$ and $p_t^2$ as a function of $E_{t-1}\ m_t$ and $E_{t-2}\ m_t$.

**8 –** Show that the model solution is given by

$$\begin{cases} p_t &=& E_{t-2}\ m_t + \frac{\phi}{1+\phi}\left(E_{t-1}\ m_t - E_{t-2}\ m_t\right) \\ y_t &=& \frac{1}{1+\phi}\left(E_{t-1}\ m_t - E_{t-2}\ m_t\right) + \left(m_t - E_{t-1}\ m_t\right) \end{cases}$$

Discuss the economic properties of the solution.

**The model with fixed prices and rational expectations (the Taylor model):** Assume now that prices are not only predetermined, but fixed for two consecutive periods. To get an easier solution, we slightly change the timing. In period $t$, half the agents sets the same price

$\chi_t$ for periods $t$ and $t+1$, at the expected average target level:

$$\chi_t = \frac{1}{2}(p^\star_{it} + E_t\ p^\star_{it+1})$$

while the other half sets their price in period $t+1$ for periods $t+1$ and $t+2$:

$$\chi_{t+1} = \frac{1}{2}(p^\star_{it+1} + E_{t+1}\ p^\star_{it+2})$$

The average price $p_t$ is therefore given by

$$p_t = \frac{1}{2}(\chi_{t-1} + \chi_t).$$

It is also assumed that $m$ is a random walk:

$$m_t = m_{t-1} + u_t$$

where $u$ is a white noise.

**9** – Show that the model equilibrium prices satisfy the recursion

$$\chi_t = A\,(\chi_{t-1} + E_t\ \chi_{t+1}) + (1-2A)m_t$$

where $A = \frac{1}{2}\frac{1-\phi}{1+\phi}$. Discuss this equation in economic terms.

**10** – Assume the solution for $\chi$ is of the form $\chi_t = \lambda\chi_{t-1} + (1-\lambda)m_t$. Use the previous recursion to compute the unique value of $\lambda$ with modulus smaller than one.

**11** – Show that the model solution in output is

$$y_t = \lambda y_{t-1} + \frac{1+\lambda}{2}u_t$$

Discuss the economic properties of the solution.

## II – Questions *(30 points)*

*Please propose a structured answer to each question, with as much economic content as possible. Please define the main terms and use math if needed.*

**1** – The slope of the Aggregate Supply curve.

**2** – The Consumption-Capital Asset Pricing Model.

## III – Discussion – About Gali's 1999 AER Paper (Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?) *(40 points)*

In his 1999 AER paper, Jordi Gali is estimating the following VAR:

My empirical model interprets the observed variations in (log) productivity ($x_t$) and (log) hours ($n_t$) as originating in two types of exogenous disturbances—technology and nontechnology shocks—which are orthogonal to each other, and whose impact is propagated over time through various unspecified mechanisms. That idea is formalized by assuming that the vector $[\Delta x_t, \Delta n_t]'$ can be expressed as a (possibly infinite) distributed lag of both types of disturbances:

$$(23)\quad \begin{bmatrix} \Delta x_t \\ \Delta n_t \end{bmatrix} = \begin{bmatrix} C^{11}(L) & C^{12}(L) \\ C^{21}(L) & C^{22}(L) \end{bmatrix} \begin{bmatrix} \varepsilon^z_t \\ \varepsilon^m_t \end{bmatrix} \equiv \mathbf{C}(L)\varepsilon_t$$

where $\{\varepsilon^z_t\}$ and $\{\varepsilon^m_t\}$ denote, respectively, the sequences of technology and nontechnology shocks. The orthogonality assumption (combined with a standard normalization) implies $E\varepsilon_t\varepsilon_t' = I$. Furthermore, the identifying restriction that the unit root in productivity originates exclusively in technology shocks corresponds to $C^{12}(1) = 0$. In other words, the matrix of long-run multipliers $\mathbf{C}(1)$ is assumed to be lower triangular.

The specification in (23) is based on the assumption that both productivity and hours are integrated of order one, so that first-differencing of both variables is necessary to achieve stationarity. That assumption is motivated by the outcome of standard augmented Dickey Fuller (ADF) tests which do not reject the null of a unit root in the levels of either series, but do reject the same null when applied to the first-differences (at the

[...]

**1** – Explain why it is useful to decompose the VAR innovations into two orthogonal components

**2** – Explain what are the assumptions made by Gali to get sequences of technology and non-technology shocks. Are these assumptions reasonable?

Some of the results of the estimation are given in the following table:

TABLE 1—CORRELATION ESTIMATES: BIVARIATE MODEL

| | Unconditional | Conditional | |
|---|---|---|---|
| | | Technology | Nontechnology |
| Panel A: First-differenced labor | | | |
| Hours | −0.26** | −0.82** | 0.26** |
| | (0.08) | (0.12) | (0.12) |
| Employment | −0.02 | −0.84** | 0.64** |
| | (0.07) | (0.26) | (0.13) |
| Panel B: Detrended labor | | | |
| Hours | −0.26** | −0.81** | 0.35* |
| | (0.08) | (0.11) | (0.20) |
| Employment | −0.02 | −0.35 | 0.38 |
| | (0.07) | (0.49) | (0.56) |

*Notes:* Table 1 reports estimates of unconditional and conditional correlations between the growth rates of productivity and labor input (hours or employment) in the United States, using quarterly data for the period 1948:1–1994:4. Standard errors are shown in parentheses. Significance is indicated by one asterisk (10-percent level) or two asterisks (5-percent level). Conditional correlation estimates are computed using the procedure outlined in the text, and on the basis of an estimated bivariate VAR for productivity growth and labor-input growth (Panel A) or productivity growth and detrended labor input (Panel B). Data sources and definitions can be found in the text.

**3** – Present in words the results.

**4** – Explain what is the effect of a positive technological shock on worked hours in an RBC model. What would be the typical shape of an impulse response of worked hours to a technological shock?

**5** – Think of a model with fixed price, in which aggregate demand is given by $Y = \frac{M}{P}$ and aggregate production function by $Y = AH$ where $H$ are worked hours and $A$ the technological parameter. What will be the effect of a positive technological shock $\mathrm{d}A > 0$ on $H$? What do you conclude from Gali's econometric results?