Université de Toulouse – MPSE – 2006-2007
M2 – Macroeconomics II — Fabrice Collard & Franck Portier

Final Exam

I – Problem - Business Cycles and Nominal Rigidities *(50 points)*

Consider a monetary economy with the following markets being open in each period t: a good market with price P_t, a labor market with the wage W_t and a money market. Money is the numéraire (price equals one). The economy is populated by two representative agents that behave in a competitve way: a firm and a household.

• The firm has a Cobb-Douglas technology:

$$Y_t = Z_t K_t^{\gamma} N_t^{1-\gamma} \tag{1}$$

where K_t is capital, N_t labor input, and Z_t a stochastic technological shock. It is assumed that the firm profit Π_t is distributed to the household.

Capital fully depreciates in one period so that

$$K_{t+1} = I_t \tag{2}$$

where I_t is investment in period t.

The representative household works N_t, consumes C_t in period t, and ends the period with a quantity of money M_t. He has the following preferences:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t + \omega \log \frac{M_t}{P_t} - V(N_t) \right] \tag{3}$$

where V is a convex function.

At the beginning of period t there is an aggregate stochastic multiplicative monetary shock as in Lucas (1972), denoted by μ_t. The money holdings M_{t-1} carried from the previous period are multiplied by μ_t , so that the household starts period t with money holdings $\mu_t M_{t-1}$.

The household budget constraint in period period t is :

$$C_t + \frac{M_t}{P_t} + I_t = \frac{W_t}{P_t} N_t + \kappa_t I_{t-1} + \frac{\mu_t M_{t-1}}{P_t} + \Pi_t \tag{4}$$

where κ_t is the real return in period t on capital invested in $t-1$.

The Walrasian regime (flexible price and wage) :

1 – Write down the household maximization program and derive its First Order Conditions.

2 – Write down the firm maximization program and derive its First Order Conditions.

3 – Define a competitive equilibrium of this economy

4 – Show that in equilibrium,

$$C_t = (1 - \beta\gamma) Y_t \tag{5}$$

$$I_t = K_{t+1} = \beta\gamma Y_t \tag{6}$$

$$\frac{M_t}{P_t} = \frac{\omega(1-\beta\gamma)}{1-\beta} Y_t = \chi Y_t \tag{7}$$

5 – Show that employment is constant along the Walrasian equilibrium path. Denote N this level and n its log.

6 – What is the effect of a shock μ_t on output? On prices? Discuss.

7 – Show that the walrasian nominal wage w^w and the price level are given by:

$$w_t^w = m_t + \log(1-\gamma) - \log \chi - n \tag{8}$$

$$p_t = m_t - \frac{z_t}{1-\gamma L} - \log \chi - n - \frac{\gamma \log \beta\gamma}{1-\gamma} \tag{9}$$

where L is the lag operator.

8 – Compute the correlation between the log of output and the log of the real wage. Discuss.

Nominal Wage Contracts : We now assume that the level of wages is predetermined at the beginning of each period. At this contract wage the household supplies all labor demanded by the firm. The crucial assumption is that parties to the contract aim at clearing the market ex ante (in logarithmic terms). In other terms , the contract wage will be set equal to the expected value of the Walrasian wage w^w.

9 – Compute the level of the contract wage as a function of expected money supply, conditionally on $t-1$ information.

Since the goods market clears and the firm's demand for labor is always satisfied, first order conditions of the firm are not affected.

10 – Explain why household now maximize their utility function (3) subject to the budget constraint (4), but taking N_t as *given* (and determined by firms's demands).

11 – Show that equilibrium allocations are now given by

$$\begin{aligned} y_t &= z_t + \gamma k_t + (1-\gamma) n_t & (10)\\ w_t - p_t &= y_t - n_t + \log(1-\gamma) & (11)\\ m_t &= p_t + y_t + \log \chi & (12)\\ k_{t+1} &= y_t + \log \beta\gamma & (13) \end{aligned}$$

plus the equation setting the nominal wage, as computed in question **9**.

12 – Show that

$$n_t = n + m_t - E_{t-1} m_t \tag{14}$$

Comment.

Let's define the monetary shock as

$$\varepsilon_{mt} = m_t - E_{t-1} m_t \tag{15}$$

13 – Using the expression of output and real wage (both in logs), show that supply shocks and *lagged* money shocks induce a positive correlation between real wage and output, while *contemporary* money shocks induce inversely a negative correlation between real wage and output. Discuss.

II – Questions *(30 points)*

Please propose a structured answer to each question, with as much economic content as possible. Please define the main terms and use math if needed.

1 – The limits of the Aggregate Demand/Aggregate Supply model.

2 – Discuss the quotation from Russ Cooper and Andrew John paper "Coordinating Coordination Failures in Keynesian Models", Quarterly Journal of Economics, vol 103 (August 1988), pp 441-443. Use elements of the course and small classes discussion in your answer.

> "[...] it captures the intuition that economies may get stuck at low levels of activity when agents are constrained in their sales. [...] There is a coordination problem in such economies if low-level equilibria could be avoided by a simultaneous increase in the output of all firms. However, in a decentralized system there may be no incentive for a single firm to increase production because this agent takes the actions of others as given. Hence, the "externality" is brought about by demand linkage that individual firms do not internalize.
>
> Coordination problems of this type are impossible in a Walrasian economy, where agents can sell any amount they choose at a given price. A demand externality may arise, though, in market structures where agents require information on both prices and quantities in making choices: this includes economies with imperfect competition or price rigidities. In both cases, quantities matter to individual decision makers, and prices do not completely decentralize allocation [...]."

III – Discussion – About Chari, Kehoe and McGrattan Paper ("Accounting for the Great Depression"), The *Federal Reserve Bank of Minneapolis Quarterly Review*, Spring 2003, Vol. 27, No. 2, pp. 2-8 *(40 points)*

1 – Read the extract reproduced in Table 1. Derive equations (2) to (4).

2 – Describe microfounded economic models behind each of those three wedges?

3 – The Figure in table 2 displays the measure of the first two wedges. Comment.

4 – Why is it more difficult to measure the investment wedge?

5 – Comment Chart 2 and 3 in Table 3.

Table 1: Extract from Chari, Kehoe and McGrattan [2003]

The Prototype Economy

The prototype economy is a growth model with three stochastic variables: A_t, τ_{lt}, and τ_{xt}. Using standard notation, we say that in any period t, consumers maximize expected utility over consumption c_t and labor l_t, $E_t\sum_t\beta^t U(c_t, l_t)$, subject to the economy's budget constraint:

$$c_t + (1+\tau_{xt})[k_{t+1} - (1-\delta)k_t] = (1-\tau_{lt})w_t l_t + r_t k_t + T_t \tag{1}$$

where k_t is the capital stock; w_t, the wage rate; r_t, the rental rate on capital; β^t, the discount factor; δ, the depreciation rate of capital; and T_t, lump-sum taxes; τ_{xt} and τ_{lt} are the tax rates on investment and labor, respectively. The firms' production function is $F(k_t, l_t)$, their productivity is A_t, and their aggregate output is y_t. Firms maximize $A_t F(k_t, l_t) - r_t k_t - w_t l_t$. The equilibrium is summarized by the resource constraint, $c_t + k_{t+1} = y_t + (1-\delta)k_t$, together with

$$y_t = A_t F(k_t, l_t) \tag{2}$$

$$-U_{lt}/U_{ct} = (1-\tau_{lt})A_t F_{lt} \tag{3}$$

$$(1+\tau_{xt})U_{ct} = \beta E_t U_{ct+1}[A_{t+1}F_{kt+1} + (1+\tau_{xt+1})(1-\delta)]. \tag{4}$$

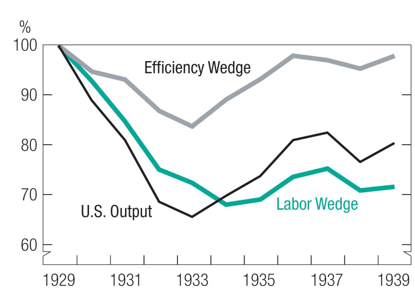
We call A_t the *efficiency wedge,* $1 - \tau_{lt}$ the *labor wedge,* and $1/(1+\tau_{xt})$ the *investment wedge*.

Table 2: Extract from Chari, Kehoe and McGrattan [2003]

Chart 1

Actual U.S. Output and Two Measured Wedges

Percentage of Each Series' Value in 1929; Annually, 1929–39



Sources of basic data: U.S. Commerce, various dates and 1975; Kendrick 1961

Table 3: Extract from Chari, Kehoe and McGrattan [2003]