Université des Sciences Sociales de Toulouse
MPSE
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DEA Macroéconomie II — Cours de Franck Portier

Final Exam

I – Problems - RBC Models *(15 points)*

Problem I-1

Consider a simple RBC model. The representative household maximizes

$$E_t \sum_{i=0}^{\infty} \beta^i u(c_{t+i}, 1-n_{t+i}),$$

where c is consumption and n is time spent in production. The household faces a budget constraint given by

$$k_{t+1} = w_t n_t + r_t k_t + (1-\delta)k_t - c_t,$$

rents capital and sells labor services to firms. Firms maximize profits, subject to a constant returns to scale technology for producing output, given by

$$y_t = e^{z_t} F(n_t, k_t),$$

where $z_t = \rho z_{t-1} + \varepsilon_t$, and $0 \leq \rho \leq 1$.

1 – Write down the equilibrium conditions for this economy (assume all markets are perfectly competitive).

2 – Assume

$$u(c_t, 1-n_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \Psi \frac{n_t^{1+\eta}}{1+\eta} \quad \text{and} \quad F(n_t, k_t) = n_t^{\alpha} k_t^{1-\alpha}.$$

For each of the unknown parameters $(\alpha, \rho, \delta, \beta, \sigma, \eta, \Psi)$, briefly discuss how you might calibrate their value.

3 – Let's consider three characteristics of actual business cycles *(i)* output displays persistent fluctuations, *(ii)* employment and output are highly correlated, *(iii)* real wages are very weakly related to output. Are there parameter values for which the model of this question can account for these business cycle "facts"? If so, are these reasonable values for the parameters (i.e., are they the ones you would obtain from the calibrations described the preceding question)? If they are not, briefly discuss how might you modify the model to better match these three stylized facts?

Problem I-2

Consider the following simple RBC model: Preferences are given by

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{(\psi_{t+i} c_{t+i})^{1-\sigma}}{1-\sigma} + \theta \log(1-n_{t+i}) \right]$$

with $0 < \beta < 1$ and $\sigma > 1$. Technology is:

$$y_t = e^{z_t} k_t^{\alpha} n_t^{1-\alpha}$$

and the resource constraint is

$$c_t + k_{t+1} - (1-\delta)k_t = y_t$$

In this setup, ψ is a "taste shock". Assume ψ has mean 1 and is serially uncorrelated.

1 – Set up the social planner's problem for this economy and derive the first order conditions. Eliminate all Lagrange multipliers so that the first order conditions only involve the utility function and/or the production function.

2 – Does ψ enter any of the equilibrium conditions? Explain intuitively how you would expect a positive realization of ψ to affect consumption and labor supply. In your explanation, be sure to explain how your intuition is consistent with the way ψ enters the model's equilibrium conditions.

3 – Do taste shocks help account for the weak correlation between real wages and employment that is observed in the data? Explain from the first order conditions and the expression of the real wage in a competitive economy.

II – Questions *(15 points)*

Please propose a structured answer to each question, with as much economic content as possible. Please define the main terms and use math if needed.

1. *The Equity Premium Puzzle.*
2. *Rational expectations and economic policy.* To illustrate your point, solve the following Aggregate Demand - Aggregate Supply model under static and rational expectations:

$$\begin{aligned} y_t &= \lambda y_{t-1} + \alpha(p_t - p_t^e) \quad (AS) \\ y_t &= -\beta p_t + \gamma m_t \quad (AD) \end{aligned}$$

where y is output, p is the price level, p^e the price expectation, that can be static ($p^e_t = p_{t-1}$) or rational ($p^e_t = E_{t-1}p_t$), m is the money supply. m_t is observed in period t.

3. *The slope of the Aggregate Supply curve.*

III – Discussion – About Lindé's Paper ("Testing for the Lucas Critique: A Quantitative Investigation"), The *American Economic Review*, Vol. 91, No. 4. (Sep., 2001), pp. 986-1005 *(15 points)*

1 – Read carefully the introduction of the paper that is reproduced in Table 1. Describe the similarity of Lindé's approach with the one taken by Cardia in her Ricardian Equivalence paper (summarize that paper) (Read also the extract of Table 2 where is found the definition of superexogeneity).

2 – Show how to compute equation (25) of the extract in Table 2. Draw the impulse response function of x to v for a high and a low value of ϕ. Compute the instantaneous multiplier in both cases. Discuss.

3 – Explain why the procedure described in Table 3 is a way of assessing the *quantitative* relevance of the Lucas critique.

4 – Explain how one should read the Table 2 of Lindé text, that is reproduced in Table 4 of this exam. How does Lindé reach the following conclusion:

> *"The conclusion is that the Lucas critique is quantitatively important in this model in a statistical sense for every regime except one, and the superexogeneity test should recognize that."*

5 – The summary of the last section of Lindé (in which is checked the power of superexogeneity tests) is

> To sum up this section, it seems to be a robust finding in this model that the test for superexogeneity has very low power in small samples—although we have given the test the best possible environment for detecting the regime shifts—and hence is not able to shed light on the practical importance of the Lucas critique in small samples.[22]

Discuss the usefulness and generality of this result. What will be your main criticism to Lindé's approach?

Testing for the Lucas Critique: A Quantitative Investigation

By JESPER LINDÉ*

In this paper, I try to shed some new light on the "puzzle" of why the Lucas critique, believed to be important by most economists, seems to have received very little empirical support. I use a real-business-cycle model to verify that the Lucas critique is quantitatively important in theory, and to examine the properties of the superexogeneity test, which is used to detect the applicability of the Lucas critique in practice. The results suggest that the superexogeneity test is not capable of detecting the relevance of the Lucas critique in practice in small samples. (*JEL* C52, C22, E41)

In a very influential article, Robert E. Lucas, Jr. (1976) raised a serious critique against econometric models that were used for policy evaluation. Lucas's argument was that shifts in economic policy change how policy affects the economy because agents in the economy are forward- rather than backward-looking and adapt their expectations and behavior to the new policy stance. Thus, past behavior can be a poor guide for assessing the effects of policy actions. For this reason, Lucas concluded that reduced-form econometric models cannot provide useful information about the actual consequences of alternative policies because the structure of the economy will change when policy changes, thereby rendering the estimated parameters in reduced-form econometric models nonconstant.

Instead, Lucas (1975, 1977), Finn E. Kydland and Edward C. Prescott (1982), and others initiated a new research program, often termed the real-business-cycle (or equilibrium-business-cycle) approach, where the models used for policy analysis are less susceptible to the Lucas critique in that they are equilibrium models with forward-looking properties. Other researchers were concerned about the applicability of the Lucas critique in practice [see, e.g., the discussion in Christopher A. Sims (1982)]. Robert F. Engle et al. (1983) introduced the concept of *superexogeneity,* and argued that it could be used to test the empirical relevance of the Lucas critique. Subsequent papers (e.g., Engle and David F. Hendry, 1993) have shown how this concept can be applied.

Recently, the empirical relevance of the Lucas critique has received increased attention. A possible explanation for this is the extensive use of backward-looking models in monetary policy analysis [cf., Laurence Ball (1999); Lars E. O. Svensson (1997); Glenn D. Rudebusch and Svensson (1999); and John B. Taylor (1999)]. Jeffrey C. Fuhrer (1997) maintains that backward-looking behavior seems to be a better approximation of reality in macroeconomic models than forward-looking behavior. Arturo Estrella and Fuhrer (1999) argue that the Lucas critique is an empirically testable hypothesis. They provide evidence that when there is a change in monetary policy regime, some forward-looking models may be less stable than their better-fitting backward-looking counterparts, which they contend is an observation inconsistent with the Lucas critique. In addition, most—if not all—of the many papers that have used the concept of superexogeneity to examine the Lucas critique empirically have found no

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evidence in favor of the proposition. See the survey by Neil R. Ericsson and John S. Irons (1995).

Two natural questions then arise. Even if the Lucas critique is qualitatively important in an equilibrium model, is it obvious that we would find the Lucas critique to be quantitatively important according to statistical criteria?[1] Second, given that the answer to the first question is yes, does the test for superexogeneity work in the sense that it reveals the presence of the Lucas critique?

In this paper, I examine these two questions in an attempt to shed some new light on the "puzzle": why is it that the Lucas critique, although regarded as highly important, does not seem to be important in studies of real-world data? My approach is to set up a version of Thomas F. Cooley and Gary D. Hansen's (1995) real-business-cycle model with money, modified to include government expenditures and a Taylor-type policy rule (see Taylor, 1993) for nominal money growth similar to that analyzed by Bennett T. McCallum (1984, 1988). This policy rule is then estimated on U.S. data for the recent periods in office of Federal Reserve's chairmen Burns, Volcker, and Greenspan. According to John P. Judd and Rudebusch (1998), the conduct of monetary policy has varied systematically between these periods.[2]

I study the dependence of two relationships on the monetary policy rule. The first relationship is a traditional money demand function [similar to those analyzed in Stephen M. Goldfeld and Daniel E. Sichel (1990)] and the second is a "Keynesian" consumption function. An important reason for focusing on money demand and consumption is that most empirical studies in the field have applied the superexogeneity test to these relations [see Ericsson and Irons (1995)].

To investigate the theoretical applicability of the Lucas critique for these relationships, I begin by deriving analytical solutions for the money demand and consumption functions to see whether the parameters are dependent on the monetary policy rule. The equilibrium model is then calibrated with different estimated monetary policy regimes, to study whether the properties of the estimated money demand and consumption functions change significantly when there is a monetary regime shift. This is done using a simple Gregory C. Chow (1960) test.

Finally, I examine the small-sample properties of the superexogeneity test. A test of superexogeneity is applied on money demand/consumption along with the monetary policy rule by means of a Monte Carlo simulation. The purpose is thus to check whether the test is actually able to identify the relevance/nonrelevance of the Lucas critique in the model economy.

One possible reason why the test may have failed to detect parameter instability in behavioral relationships, despite the presence of parameter instability in policy rules, is that the effects of changes in policy rules are very difficult to distinguish from other shocks that hit the economy. However, in the general-equilibrium framework used here, it is in fact possible to control these effects by "going back in time" and performing the superexogeneity test conditional on all other shocks, except for the change in the monetary policy rule.

The results of the paper are as follows. First, it is demonstrated that in the equilibrium model, the parameters in the money demand and consumption functions are functions of the parameters in the monetary policy rule.[3] Consequently, the Lucas critique is, as expected, at

[1] For a discussion along this line, see Eric M. Leeper's (1995) comments on the paper by Ericsson and Irons (1995).

[2] Judd and Rudebusch (1998) start out by noting that there is instability in the Fed reaction function. They then find support for the hypothesis that the Fed monetary policy rule has varied systematically with the different periods in office of Fed chairmen Burns, Volcker, and Greenspan. As in their analysis, chairman Miller is omitted here because of his very short tenure.

[3] As regards money demand, the results both contradict and support the proposition in Lucas (1988) that money demand is a structural relation (invariant to policy parameters). The "true" money demand function in the general-equilibrium model is independent of parameters in the monetary policy rule as suggested by Lucas. However, the "true" money demand function derived in the model is not in the form typically estimated by economists. Moreover, when it is rewritten in a traditional form [as in Goldfeld and Sichel (1990)], the reduced form parameters turns out to be dependent of the monetary policy rule. Robert G. King (1988) discusses this issue.

least qualitatively important in the model. Second, with a standard parameterization of the model and by considering the estimated Federal Reserve monetary policy rules for nominal money growth during the Burns, Volcker, and Greenspan office periods, it is found that the Lucas critique is theoretically important in a statistically significant way. When the parameters in the estimated Taylor-type monetary policy rule change, the estimated money demand and consumption functions display considerable parameter instability from both a statistical and an economic point of view. I thus have a model where the superexogeneity test should be able to identify the empirical relevance of the Lucas critique.

Despite this, it is found that the superexogeneity test far too often fails to reject the false null hypothesis that the Lucas critique does not apply when there is a change in the conduct of monetary policy. This lack of power for the superexogeneity test is then, quite naturally, a possible explanation for why the Lucas critique has not been found in the data.

The findings of this paper also have some general implications for empirical testing of the relevance of backward- versus forward-looking models. First, only the truly forward-looking model will have parameters invariant to the monetary regime. Thus, the preliminary results in Estrella and Fuhrer (1999), suggesting that the Lucas critique is not important in practice, may be attributed to model misspecification. Second, these types of tests will presumably have very weak power in small samples, as is the case for the superexogeneity test.

The structure of the paper is as follows. In Section I I describe the theoretical model and indicate how to compute the equilibrium. Estimation and calibration issues are addressed in Section II. In Section III I derive and discuss the theoretical properties of the money demand and consumption functions used throughout the paper. Next, in Section IV I examine whether the Lucas critique is significantly important in the equilibrium-business-cycle model by testing for parameter stability between the different estimated monetary policy regimes on simulated data from the model. The superexogeneity test is briefly presented in Section V, along with the results of the Monte Carlo simulations. My conclusions are presented in Section VI.

Table 2: Extract from Lindé 2001

A. *Superexogeneity: Concept, Testing, and a Formal Definition*

Consider the following simple (presented in reduced form) forward-looking model in the spirit of Lucas (1976):

$$(24) \quad x_t = \theta \mathrm{E}_t \sum_{j=0}^{\infty} z_{t+j} + \varepsilon_t,$$

$$\varepsilon \sim \text{i.i.d. } N(0, \sigma_\varepsilon^2),$$

$$z_t = \phi z_{t-1} + v_t,$$

$$-1 < \phi < 1, \qquad v \sim \text{i.i.d. } N(0, \sigma_v^2).$$

In (24), z_t is the policy variable and x_t the target variable. Solving (24) for x_t as a function of z_t, we obtain

$$(25) \qquad x_t = \beta_z z_t + \varepsilon_t,$$

where $\beta_z \equiv \theta/(1 - \phi)$. If the econometrician estimates (25), ignoring the dependency of β_z on ϕ, then policy simulations based on the estimate $\hat{\beta}_z$ for alternative paths of $\{z_{t+j}\}_{j=0}^{\infty}$ (treating v_{t+j} as a fixed exogenous shock), and thus for alternative paths of ϕ, will give misleading results.

Testing for the constancy/nonconstancy of ϕ and β_z in (24) and (25) by estimating these equations then provides a simple way of testing for the Lucas critique; if β_z is constant but ϕ is not, then the Lucas critique cannot apply. z_t is then said to be superexogenous for β_z.

Table 3: Extract from Lindé 2001

The procedure in the simulations has been as follows:

1. Simulate the model for T periods under the assumption that the monetary policy rule changes completely unexpectedly after $T/2$ periods from one regime to another (for example, from Burns to Volcker or Burns to Greenspan).[13]
2. Estimate the money demand equation (21), consumption function (23), and monetary policy rule (8) with OLS on the first $1, \ldots, T/2$ observations in the simulated sample. Denote the estimated parameter vectors $\hat{\boldsymbol{\beta}}_{MD}$, $\hat{\boldsymbol{\beta}}_{CF}$, and $\hat{\boldsymbol{\beta}}_{TR}$, respectively.
3. Estimate (21), (23), and (8) with OLS on the last $T/2 + 1, \ldots, T$ observations in the simulated sample. Denote the estimated parameter vectors $\hat{\boldsymbol{\alpha}}_{MD}$, $\hat{\boldsymbol{\alpha}}_{CF}$, and $\hat{\boldsymbol{\alpha}}_{TR}$, respectively.
4. Use a version of the F-test, often called the Chow breakpoint test, to examine whether the null hypotheses $\boldsymbol{\alpha}_{MD} = \boldsymbol{\beta}_{MD}$, $\boldsymbol{\alpha}_{CF} = \boldsymbol{\beta}_{CF}$, and $\boldsymbol{\alpha}_{TR} = \boldsymbol{\beta}_{TR}$ are rejected at the 5-percent significance level.
5. Repeat Steps 1–4 many (N) times to compute probabilities for how often the null hypotheses are rejected for the given significance level.
6. To get correct significance levels, Steps 1–5 are carried out twice. In the first round, small-sample critical values are computed under the (true) null hypotheses $H_0 : \boldsymbol{\alpha}_{MD} = \boldsymbol{\beta}_{MD}$, $H_0 : \boldsymbol{\alpha}_{CF} = \boldsymbol{\beta}_{CF}$, and $H_0 : \boldsymbol{\alpha}_{TR} = \boldsymbol{\beta}_{TR}$ (that is, compute the distribution of F-statistics when there has been no regime shift). In the second round, these adjusted critical values ensure a correct size in the F-testing for regime shifts.

If the computed probabilities in Step 5 (in the second round) of rejecting parameter stability are lower/higher than the given significance levels, the Lucas critique is/is not relevant in this model in a statistical sense.

Table 4: Extract from Lindé 2001

TABLE 2—CHOW TEST PROBABILITIES OF REJECTING THE NULL HYPOTHESIS OF PARAMETER STABILITY IN MONEY DEMAND, CONSUMPTION, AND MONETARY POLICY RULE AT THE 5-PERCENT SIGNIFICANCE LEVEL

	T = 100 Comparison regime				T = 200 Comparison regime			
Benchmark regime	WS	B	V	G	WS	B	V	G
				The money demand function (21)				
Whole sample (WS)	0.050	0.219	0.083	0.046	0.050	0.416	0.121	0.044
Burns (B)	0.654	0.050	0.328	0.463	0.916	0.050	0.711	0.785
Volcker (V)	0.422	0.319	0.050	0.212	0.607	0.671	0.050	0.330
Greenspan (G)	0.092	0.208	0.062	0.050	0.101	0.440	0.088	0.050
				The consumption function (23)				
Whole sample	0.050	0.277	0.123	0.044	0.050	0.258	0.116	0.041
Burns	0.462	0.050	0.164	0.331	0.708	0.050	0.438	0.596
Volcker	0.502	0.354	0.050	0.312	0.708	0.673	0.050	0.513
Greenspan	0.080	0.283	0.096	0.050	0.084	0.352	0.114	0.050
				The Taylor-type rule for monetary policy (8)				
Whole sample	0.050	0.863	0.606	0.078	0.050	0.999	0.941	0.093
Burns	0.943	0.050	0.258	0.687	1.000	0.050	0.517	0.975
Volcker	0.584	0.154	0.050	0.209	0.944	0.365	0.050	0.469
Greenspan	0.073	0.481	0.209	0.050	0.100	0.926	0.465	0.050

Notes: The diagonals equal 0.050 because the critical values used in the testing were simulated under the null hypothesis of no-regime shift (i.e., $H_0 : \boldsymbol{\alpha}_{MD} = \boldsymbol{\beta}_{MD}$, $H_0 : \boldsymbol{\alpha}_{CF} = \boldsymbol{\beta}_{CF}$, and $H_0 : \boldsymbol{\alpha}_{TR} = \boldsymbol{\beta}_{TR}$ are true). The Chow (1960) statistic