Université des Sciences Sociales de Toulouse
MPSE
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DEA Macroéconomie II — Cours de Franck Portier

Final Exam

I – Problem – RBC, Labor Market and Bonds Market *(12 points)*

We consider here a simple analytical RBC model, and study some implications for labor and bonds markets. The economy is populated with a representative household and a representative firm. The firm has a Cobb-Douglas technology:

$$Y_t = \Theta_t K_t^{\gamma} L_t^{1-\gamma} \tag{1}$$

where K_t is capital, L_t labor input, and Θ_t the stochastic Total Factor Productivity (TFP). One assumes $\Theta_t = e^{\varepsilon_t}$, where ε is a white noise with variance σ_ε^2. All profits of the firm are distributed to the household. Capital evolves according to

$$K_{t+1} = I_t \tag{2}$$

where I_t is investment in period t.

The representative household works L_t and consumes C_t. Preferences are given by

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - \mu \Lambda_t L_t^{\frac{1}{\mu}}\right] \tag{3}$$

where $\Lambda_t = e^{\nu_t}$ is a preference shock, with ν being white noise with variance σ_ν^2. Capital is accumulated by the household and rented to the firm.

Let the final good be the numéraire, κ be the real rental rate of capital and w the real wage.

1 – Write down the household maximization program and derive its First Order Conditions.

2 – Write down the firm maximization program and derive its First Order Conditions.

3 – Define a competitive equilibrium of this economy

4 – Solve the model to obtain $y_t = \varepsilon_t + \gamma y_{t-1} - (1-\gamma)\mu\nu_t$ (dropping constants and with the notation $x = \log X$)

5 – Compute the unconditional variance of y, c and i. Comment

6 – Compute the correlation between the real wage w and output y when $\sigma_\nu^2 = 0$. Is it in line with what we find in the data? What does become this correlation when $\sigma_\nu^2 > 0$?

7 – Use the household first order conditions to write a log-linear labor supply function (for a given consumption c_t) and the firm ones to derive a labor demand function. Assuming constant consumption, illustrate using graphs the effect of ε and ν on the w, ℓ and w, y correlation.

8 – Compute the price P_t of a bond that pays 1 unit of goods with probability one in period $t+1$. Use the approximation $\log E_t\left(\frac{1}{C_{t+1}}\right) \approx -c_{t+1}$ (with again the small letter being the log of the capital ones) to compute and comment the sign of $\frac{\partial P_t}{\partial \varepsilon_t}$ and $\frac{\partial P_t}{\partial \nu_t}$.

II – Questions *(12 points)*

Please propose a structured answer to each question, with as much economic content as possible. Please define the main terms and use math if needed.

1. The role of capital accumulation in RBC models.
2. What do recent episodes of fiscal adjustment in the OECD tell us about Ricardian Equivalence (to be defined)?
3. The construction of the Aggregate Demand Curve.

III – Discussion – About Jermann's 1998 JME Paper ("Asset Pricing in Production Economies") *(12 points)*

1 – What did Mehra and Prescott 1985 tell us about asset pricing in an endowment economy? (explain what they did and what they obtained)

2 – The text in Table 1 is taken from the introduction of Jermann's paper. Explain why capital adjustment costs and habit persistence are needed to solve the equity premium puzzle?

3 – How does one derive equation (3.2) in Table 2?

4 – Comment Table 3.

Table 1: Extract from the introduction of Jermann 1998

One line of progress for solving the equity premium puzzle has been to modify preferences and payout structures for the case where consumption is specified so as to replicate aggregate data. Most of these studies use the endowment economy framework.[2] However, attempts to explain the equity premium in models with nontrivial production sectors, that is, models where consumption and dividends have to be derived endogenously, were less successful (e.g. Danthine et al., 1992; Rouwenhorst, 1995).[3] To some extent, it should not be too surprising that the difficulty for a general equilibrium model to explain asset returns is increased when consumption and dividends also have to be derived endogenously. For instance, Rouwenhorst (1995) finds that it is more difficult to explain substantial risk premia because endogenous consumption becomes even smoother as risk aversion is increased. The reason behind this is that in the standard one-sector model agents can easily alter their production plans to reduce fluctuations in consumption. This suggests that the frictionless and instantaneous adjustment of the capital stock is a major weakness in this framework. One way to reduce consumption smoothing through the production sector, is to introduce capital adjustment costs. Capital adjustment costs have a long tradition in the investment literature, they also provide a formal framework for the popular 'q' theory (q is defined as the value of the capital stock divided by its replacement cost). It therefore seems natural to introduce capital adjustment costs into this standard framework. In fact, without capital adjustment costs, as most current business cycle models are, these models are plagued by a counterfactual constant q.

Given its previous success in solving the equity premium puzzle in models with trivial production sectors (e.g. Abel, 1990; Constantinides, 1990) our analysis also includes habit formation preferences, in addition to the standard time-separable specification. We can thus study how these preferences fare in general equilibrium when required to jointly explain asset returns and business cycles.

We find that a real business cycle model – by replicating the basic business cycle facts – can generate the historical equity premium with *both* capital adjustment costs and habit formation, but not with either taken separately. The main reason why this combination is successful is quite intuitive: with no habit formation, marginal rates of substitutions are not very volatile, since people do not care very much about consumption volatility; with no adjustment costs, they choose consumption streams to get rid of volatility of marginal rates of substitution. They have to both care, and be prevented from doing anything about it.[4]

Table 2: Extract from Jermann 1998

The second step of our solution method is to apply lognormal pricing formulae following Hansen and Singleton (1983), Campbell (1986) and Campbell (1996). The basic asset pricing formula uses the fact that any claim to a potentially random future payout $D_{t+k}(s_{t+k})$ (for dividend) can be valued by the present value relationship:

$$V_t^{D_k}(s_t) = \frac{\beta^k \mathrm{E}_t[\Lambda_{t+k}(s_{t+k}) D_{t-k}(s_{t-k})]}{\Lambda_t(s_t)}, \tag{3.2}$$

where β is the pure time discount factor, and $\Lambda_{t+k}(s_{t+k})$ the marginal valuation (or marginal utility) of the numeraire at $t+k$.

Table 3: Extract from Jermann 1998

Business cycles and asset returns

Model version/Moments	$\sigma_{\Delta C}/\sigma_{\Delta Y}$	$\sigma_{\Delta I}/\sigma_{\Delta Y}$	$\mathrm{E}(r^f)$	$\mathrm{E}(r^e - r^f)$	$\mathrm{Std}(r^f)$	$\mathrm{Std}(r^e)$	$\mathrm{E}(r^b - r^f)$
Benchmark	0.49	2.64	0.82	6.18	11.46	19.86	5.69
Standard RBC model (No habit, no adj. costs)	0.77	1.54	4.26	0.02	0.62	1.02	0.04
Risk aversion = 10, no habit, no adj. costs	0.78	1.53	3.36	0.26	0.76	2.90	0.29
Habit, no adjustment costs	0.33	3.00	4.20	0.03	0.59	1.21	0.08
Adjustment cost, no habit	1.14	0.68	3.91	0.67	0.61	6.09	0.45
Random walk productivity	0.55	2.57	0.03	6.39	11.98	18.80	5.09
Data	0.51	2.65	0.80	6.18	5.67	16.54	1.70

The symbols have the following meaning: $\sigma_{\Delta Y}$, standard deviation of quarterly output growth rates; $\sigma_{\Delta C}$, standard deviation of quarterly consumption growth rates; $\sigma_{\Delta I}$, standard deviation of quarterly investment growth rates; r^f, risk-free interest rate; r^e, return to equity; r^b, return to a perpetual bond in model, long-term government bond in the data. Business cycle growth rate data is from the NIPA, 54.1–89.2, GNP for output, Consumption of nondurables and services for consumption, Fixed investment for investment. Equity and short-term bond returns are from Mehra and Prescott (1985) long term government bond returns are from Ibbotson (1994). Business cycle data is quarterly and asset return data is annualized, both are in percentage terms. Business cycle data and risk-free rates are (computed) population moments, the remaining asset return moments are averages of 100 simulations each 200 periods long.