Université des Sciences Sociales de Toulouse
MPSE
Année universitaire 2002-2003
DEA Macroéconomie II — Cours de Franck Portier

Final Exam

I – Problem – Credit Controls in an Overlapping-Generations Economy *(50%)*

Consider the following overlapping-generations model. At each date $t \geq 1$ there appear N two-period-lived young people, said to be of generation t, who live and consume during periods t and $(t+1)$. At time $t = 1$ there exist N old people who are endowed with $H(0)$ units of paper, "euros", which they offer to supply inelastically to the young of generation 1 in exchange for goods. Let $p(t)$ be the price of the one good in the model, measured in euros per time t good. For each $t \geq 1$, $N/2$ members of generation t $(h = 1, ..., N/2)$ are endowed with $w_t^h(t) = y > 0$ units of the good at t and $w_t^h(t+1) = 0$ units at $(t+1)$, whereas the remaining $N/2$ members of generation t $(h = N/2+1, ..., N)$ are endowed with $w_t^h(t) = 0$ units of the good at t and $w_t^h(t+1) = y > 0$ units when they are old. All members of all generations have the same utility function:

$$u[c_t^h(t), c_t^h(t+1)] = \log c_t^h(t) + \log c_t^h(t+1)$$

where $c_t^h(s)$ is the consumption of agent h of generation t in period s. The old at $t = 1$ simply maximize $c_0^h(1)$. The consumption good is nonstorable. The currency supply is constant through time, so $H(t) = H(0)$ for all $t \geq 1$. The real interest rate on loans is denoted by $r(t)$.

1. Write down the program faced by the young generation of period t, denoting $m_t^h(t)$ the level of nominal money holding and $l_t^h(t)$ the level of claims on $(t+1)$-period consumption purchased (if positive) or sold (if negative) by household h of generation t.

2. Explain why such a model is likely to posses a monetary and a non-monetary steady state.

3. Define a competitive equilibrium without valued currency for this model. Who trades what with whom?

4. Compute the individual saving function $s^h(t)$. Derive the aggregate saving function $f[1 + r(t)]$.

5. Compute the nonvalued-currency competitive equilibrium values of the interest rate, the consumption allocation of the old at $t = 1$, and that of the "borrowers" and "lenders" for $t \geq 1$. *Hint: Think of what should be the aggregate level of savings at a non-monetary equilibrium*

6. Define a competitive equilibrium with valued currency. Who trades what with whom?

7. Prove that for this economy there does not exist a competitive equilibrium with valued currency. *Hint: Derive an arbitrage condition between money and loans from the typical household first order conditions, and use it together with the aggregate saving function and the good market equilibrium condition.*

8. Now suppose that the government imposes the restriction that $(1 + r(t))l_t^h(t) \geq -y/4$, where $l_t^h(t)$ represents claims on $(t+1)$-period consumption purchased (if positive) or sold (if negative) by household h of generation t. This is a restriction on the amount of borrowing. For an equilibrium without valued currency, compute the consumption allocation and the real interest rate.

9. In the setup of question 8, show that there exists a stationary equilibrium with valued currency in which the price level obeys the quantity theory equation $p(t) = qH(0)/N$. Find a formula for the undetermined coefficient q. Compute the consumption allocation and the equilibrium rate of return on consumption loans.

10. Are lenders better off in economy of question 5 or economy of question 9? What about borrowers? What about the old of period 1 (generation 0)?

11. What do we learn from this model?

II – Questions *(30%)*

Please propose a structured answer to each question, with as much economic content as possible. Please define the main terms and use math if needed.

1. Why should we care about the slope of the Aggregate Supply curve (to be defined)?

2. Technological shocks in RBC models.

3. The optimum quantity of Money and the Friedman Rule.

III – Text Discussion – About Lucas' 1973 Paper (Some International Evidence on Output-Inflation Tradeoffs, AER 1973) *(20%)*

1. What is the objective pursued by Lucas in his 1973 paper?

2. Describe in words the assumptions and results of Lucas' model

3. The following text is an extract from Lucas' paper. Comment those results. How do they confirm/infirm Lucas' view?

"In terms of ΔP_t and $y_{c,t}$, and letting $\pi = \theta\gamma/(1+\theta\gamma)$, the solutions are

$$y_{ct} = -\pi\delta + \pi\Delta x_t + \lambda y_{c,t-1} \tag{11}$$

$$\Delta P_t = -\beta + (1-\pi)\Delta x_t + \pi\Delta x_{t-1} - \lambda\Delta y_{c,t-1} \tag{12}$$

[*Recall that in this paper, y_c is cyclical (real) output, x is nominal output, that the individual supply curve in island z is $y_{ct}(z) = \gamma[P_t(z) - E(P_t|I_t(z))] + \lambda y_{c,t-1}(z)$, that $\theta = \tau^2/(\tau^2+\sigma^2)$ and that τ^2 is the variance of the idiosynchratic noise z and σ^2 the variance of P_t is the equation $P_t(z) = P_t + z$.*]

[...]

Descriptive statistics for the eighteen countries in the sample are given in Table 1.

[...]

The first three columns of Table 2 summarize the performance of equation (11) in accounting for movements in y_{ct}. [...] The R^2s for the inflation rate equation (12) are given in column (4) of Table 2 [...] Column (5) of TAble 2 gives the fraction of the variance of ΔP_t explained by (12) when the coefficient estimates from (11) are imposed. (A "–" indicates a negative value.) "

Table 1 – Descriptive Statistics, 1952-67

Country	Mean Δy_t	Mean ΔP_t	Variance y_{ct}	Variance ΔP_t	Variance Δx_t
Argentina	.026	.220	.00096	.01998	.01555
Austria	.048	.038	.00104	.00113	.00124
Belgium	.034	.021	.00075	.00033	.00072
Canada	.043	.024	.00109	.00018	.00139
Denmark	.039	.041	.00082	.00038	.00084
West Germany	.056	.026	.00147	.00026	.00073
Guatemala	.046	.004	.00111	.00079	.00096
Honduras	.044	.012	.00042	.00084	.00109
Ireland	.025	.038	.00139	.00060	.00111
Italy	.053	.032	.00022	.00044	.00040
Netherlands	.047	.036	.00055	.00043	.00101
Norway	.038	.034	.00092	.00033	.00098
Paraguay	.054	.157	.00488	.03192	.03450
Puerto Rico	.058	.024	.00205	.00021	.00077
Sweden	.039	.036	.00030	.00043	.00041
United Kingdom	.028	.034	.00022	.00037	.00014
United States	.036	.019	.00105	.00007	.00064
Venezuela	.060	.016	.00175	.00068	.00127

Table 2 – Summary Statistics by Country, 1953-67 *(I have not reported T-Stats)*

Country	π	λ	R^2_y	$R^2_{\Delta P}$	R^2_ω
Argentina	.011	-.126	.018	.929	.914
Austria	.319	.703	.507	.518	–
Belgium	.502	.741	.875	.772	.661
Canada	.759	.736	.936	.418	–
Denmark	.571	.679	.812	.498	.282
West Germany	.820	.784	.881	.130	–
Guatemala	.674	.695	.356	.016	–
Honduras	.287	.414	.274	.521	.358
Ireland	.430	.858	.847	.499	.192
Italy	.622	.042	.746	.934	.914
Netherlands	.531	.571	.711	.627	.580
Norway	.530	841	.893	.633	.427
Paraguay	.022	.742	.568	.941	.751
Puerto Rico	.689	.1.029	.939	.419	–
Sweden	.287	.584	.525	.648	.405
United Kingdom	.665	.178	.394	.266	.115
United States	.910	.887	.945	.571	.464
Venezuela	.514	.937	.755	.425	–