Université des Sciences Sociales de Toulouse
MPSE
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DEA Macroéconomie II — Cours de Franck Portier

Final Exam

I – Problem – Deficits, Inflation and Anticipated Monetary Shocks

Consider an economy populated by a large number of identical individuals. Preferences over consumption and leisure are given by

$$\sum_{t=0}^{\infty} \beta^t c_t^{\alpha} \ell_t^{1-\alpha}$$

where $0 < \alpha < 1$. Assume that leisure is positively related – this is just a reduced form of a shopping time model– to the stock of real money balances, and negatively related to consumption as a measure of transactions

$$\ell_t = A \frac{m_{t+1}}{p_t} c_t^{-\eta} \quad A > 0$$

Each individual owns a Lucas tree that drops y units of consumption per period (dividends). There is a government that issues one period real bonds (promises to deliver $1/R_t$ unit of good in $t+1$, at a current cost 1), money, and collects taxes (lump-sum τ) to finance spending. Per capita spending is equal to g. Thus, consumption equals $c = y - g$. The government's budget constraint is:

$$g_t + B_t = \tau_t + \frac{B_{t+1}}{R_t} + \frac{(M_{t+1} - M_t)}{p_t}$$

Let the rate of return on money be $R_{mt} = p_t / p_{t+1}$. Let the nominal interest rate at time t be $1 + i_t = R_t p_{t+1}/p_t = R_t \pi_t$.

1. Derive the demand for money, and show that it decreases with the nominal interest rate
2. Suppose that government policy is such that $g_t = g$, $B_t = B$, $M_t = M$ and $\tau_t = \tau$. Prove that the real interest rate, R, is constant and equal to the inverse of the discount factor.
3. Define the deficit as d, where $d = g + (B/R)(R - 1) - \tau$. Write money demand as $f(R_m)$. What is the highest possible deficit that can be financed in this economy? An economist claims that –in this economy–increases in d, which leave g unchanged, will result in increases in the inflation rate. Discuss this view.
4. Suppose that the economy is open to international capital flows and that the world interest rate is $R^{\star} = \beta^{-1}$. Assume that $d = 0$, and that $M_t = M$. At $t = T$, the government increases the money supply to $M' = (1 + \mu)M$. This increase in the money supply is used to purchase bonds (government bonds). This, of course, results in a smaller deficit at $t > T$. (In this case, it will result in a surplus). However, the government also announces its intention to cut taxes (starting at $T+1$) to bring the deficit back to zero. Argue that this open market operation will have the effect of increasing prices at $t = T$ by μ%: $p' = (1 + \mu)p$, where p is the price level from $t = 0$ to $t = T - 1$.
5. Consider the same setting as in (4) Suppose now that the open market operation is announced at t = 0 (it still takes place at t = T). Argue that prices will increase at t = 0 and that, in particular, the rate of inflation between T-1 and T will be less than $1 + \mu$.

Solution of the problem

1. The Lagrangian of the problem is, replacing ℓ by its expression in term of m/p

$$\max_{c_t, b_{t+1}, m_{t+1}} \sum_{t=0}^{\infty} \beta^t c_t^{\alpha-(1-\alpha)\eta} A^{1-\alpha} \left(\frac{m_{t+1}}{p_t}\right)^{1-\alpha}$$

s.t.

$$c_t + \frac{m_{t+1}}{p_t} + \frac{b_{t+1}}{R_t} \leq y + \frac{m_t}{p_t} + b_t - \tau_t \qquad (\beta^t \lambda_t)$$

The FOCs are, wrt c_t, b_{t+1} and m_{t+1} respectively

$$(\alpha - (1-\alpha)\eta) \frac{c_t^{\alpha-(1-\alpha)\eta} A^{1-\alpha} \left(\frac{m_{t+1}}{p_t}\right)^{1-\alpha}}{c_t} = \lambda_t \qquad (1)$$

$$\frac{\lambda_t}{R_t} = \beta \lambda_{t+1} \qquad (2)$$

$$(1-\alpha)\frac{c_t^{\alpha-(1-\alpha)\eta}A^{1-\alpha}\left(\frac{m_{t+1}}{p_t}\right)^{1-\alpha}}{m_{t+1}}=\beta\frac{\lambda_{t+1}}{p_{t+1}}+\frac{\lambda_t}{p_t} \tag{3}$$

Using (2), (3) can be written

$$(1-\alpha)\frac{c_t^{\alpha-(1-\alpha)\eta}A^{1-\alpha}\left(\frac{m_{t+1}}{p_t}\right)^{1-\alpha}}{m_{t+1}}=\frac{\lambda_t}{p_t}\left(\frac{i_t}{1+i_t}\right) \tag{4}$$

and (1)/(4) gives, using $c_t=y-g_t$

$$\frac{m_{t+1}}{p_t}=\phi(y-g_t)\left(1+\frac{1}{i_t}\right) \tag{5}$$

with $\phi=\frac{1-\alpha}{\alpha-(1-\alpha)\eta}$. (5) is the money demand equation, that can also be written as

$$\frac{m_{t+1}}{p_t}=f(R_{mt})=\phi(y-g_t)\frac{R_t}{R_t-R_{mt}} \tag{6}$$

2. The BC of the govt writes

$$\left(1-\frac{1}{R_t}\right)B=\tau-g$$

and therefore R_t is constant. From (2), we obtain that $\frac{\lambda_{t+1}}{\lambda_t}=R\beta$ is constant. Let us write (1) as $\lambda_t p_t^\alpha=K$ where K is a constant. Now writing equation (1) at period t, at period $t+1$, and taking the ratio of those two equations, we get

$$\frac{\lambda_t p_t^\alpha}{\lambda_{t+1}p_{t+1}^\alpha}=1 \tag{7}$$

Therefore, as $\frac{\lambda_t}{\lambda_{t+1}}$ is constant, $\frac{p_t}{p_{t+1}}$ is constant, and so is R_m. Now take the money demand equation (6), and use the fact that at an equilibrium, $b=B$ and $m=M$. If m and R_m are constant, then p is constant. Then, from (7), we get that λ is constant. If λ is constant, then $R\beta=1$, or $R=\beta^{-1}$. *QED*

3. In this economy, the BC of the gvt gives

$$d=g-B\frac{R-1}{R}-\tau=\frac{\Delta M}{p}=0$$

Therefore, no deficit can be sustained because there is no permanent money creation, as ΔM is constrained to be constant.

Assume that money creation is possible (ΔM positive and constant). Then the deficit equation is given by

$$d=\frac{\Delta M_t}{p_t}=\frac{M_{t+1}}{p_t}-\frac{M_t}{p_t}=f(R_m)-\frac{M_t}{p_{t-1}}\frac{p_{t-1}}{p_t}=(1-R_m)f(R_m)=\Psi(R_m)$$

with $f(R_m)=\frac{\phi(y-g)}{1-\beta R_m}$. It is easy to check that $\Psi'<0$. Therefore, higher deficit means lower R_m or equivalently higher inflation. If inflation is infinite, then $R_m=0$ and the maximum deficit is $d^{\max}=f(0)=\phi(y-g)$.

4. Before period T, we have $R_m=1$, and from the money demand, $p=\frac{M}{f(1)}$. After period $T+1$ (included), we have again $R_m=1$ and $p'=\frac{M'}{f(1)}=(1+\mu)\frac{M}{f(1)}=(1+\mu)p$. Therefore, $\frac{p'}{p}=(1+\mu)$.

5. Before period 0, we have $p=\frac{M}{f(1)}=\frac{M(1-\beta)}{\phi(y-g)}$. After period $T+1$ (included), we have $p'=\frac{M'}{f(1)}=\frac{M'(1-\beta)}{\phi(y-g)}=(1+\mu)p$. Therefore, the total cumulated inflation from 0 to $T+1$ will be μ%.

Let us now consider period T. Money demand gives $p_T=\frac{M'}{f(p_T/p')}$, which gives us the value of p_T. Therefore, the price level at $T-1$ will be given by $p_{T-1}=\frac{M}{p_T/p_{T-1}}$, which can be solved for p_{T-1}, given that we have computed p_T.

Now, for every $t\in[1,T-2]$, we have the difference equation

$$p_t=\frac{M}{f(p_t/p_{t-1})} \tag{8}$$

Given the terminal condition p_{T-1}, it is possible to compute p_0, which is greater than p, because p is a fixed point of (8). (see figure 1). As inflation is always positive and sum to μ from 0 to T, it has to be strictly smaller than μ at period 0.

Figure 1: The price dynamics equation (8)