Location decisions and Minimum Wages

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Abstract

Our paper contributes to the living debate on the controversial of labor market policies on macroeconomic performances. It sheds light on an original transmission mechanism through which labor market policies affect output and employment that relies on the endogenous entry decisions of firms.

The paper incorporates features from the New Economic Geography (Krugman (1991)) and the labour market literature. In a two-country framework, we model both endogenous entry of firms and nominal rigidities. In this setting, we analyze the impact of a unilateral increase in one country’s minimum wage. We find that, for reasonable parameters values, the attractiveness of a larger demand in the home country can entice firms to come and produce locally, despite higher marginal costs. As a result, aggregate income raises and the unemployment level for low-skilled workers decreases.

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1 Introduction

The impact of labor market rigidities on macroeconomic performances lies at the heart of current European debates, among economists as well as politicians. Indeed, European continental economies distinguish themselves by labor market policies characterized by high minimum wages and generous unemployment insurance systems. Furthermore, the analysis according to which technological changes are skill-biased at the expense of low-skilled workers is commonly shared within economists (Barro and Sala-I-Martin (1995), Mincer (1995)), and their adverse effects on low-skilled workers have led labor market policies to increase protection towards this labour market segment. As recalled by CSERC (1999), the existence of a legal minimum wage is one of the key instruments of these low-skilled workers oriented policies.

Those institutional arrangements are often argued to put a brake upon economic growth: their positive impact on labor costs is considered as an impediment in European firms' competitiveness in world markets and a barrier for foreign direct investment. The efficiency of these labour market policies is all the more questioned in the current context where the globalization of the production process and the increasing competition from emerging markets makes the question of the international competitiveness of national firms more crucial.

Such an analysis entirely focuses on the effect of labor rigidities on the cost competitiveness of European firms. On the other hand, new theories of the international trade emphasize the impact of market potentials on equilibrium productive patterns (see Baldwin, Forslid, Martin, Ottaviano, and Robert-Nicoud (2005)): confronted to increasing returns and costly international trade, firms have an incentive to locate in the largest market, in terms of demand. As a consequence, firms location decisions are influenced by the cost side as well as demand determinants, whereas the latter are often neglected in public debates. Yet, the demand effect must be considered when asking for the economic performances of existing Welfare States. Indeed, the will to ensure a minimum income in European labor markets reflects the choice to maintain the purchasing power of the most vulnerable (unskilled) workers through the taxation of others. These institutional arrangements thus influence the demand pattern within a country and, potentially, the productive pattern in a New Trade framework.

The objective of the paper is to provide a tractable framework putting in evidence this contradictory influence of labor rigidities on the productive pattern. More precisely, we focus on the influence of exogenously set minimum wages on firms location decisions in a two country framework with endogenous entry of firms. Our framework sheds light
on two opposite effects of an unilateral increase in one country’s minimum wage. First, the direct cost effect pushes firms to locate in the low minimum wage country, while the indirect demand effect entices firms to produce close to workers with the highest purchasing power\(^1\). Using analytical reasoning as well as numerical simulations, we are able to distinguish situations where the cost effect dominates, in what case a minimum wage increase deters firms to enter the market, from situations where the generosity of unemployment benefits and high minimum wages attracts firms through the home market effect.

This paper thus stands between two distinct literatures. First, the impact of minimum wages is a recurrent theme in the labour literature (Cahuc and Zylberberg (1999), Dolado, Felgueroso, and Jimeno (2000), Bourguignon and Bureau (1999)). However, those papers generally assume a fixed number of firms, therefore neglecting the endogeneity of location decisions and the home market effect. On the other hand, papers in the strand of the new economic geography literature (Krugman (1991)) mainly assume perfectly flexible labour markets, which is not well-suited to the analysis of continental European labor markets. As already argued by Strauss-Kahn (2005), considering nominal rigidities may seem more appropriate for those countries, especially for the low-skilled workers segments. By incorporating both endogenous entry of firms and nominal rigidities, our framework delivers interesting results that contrast the widespread view that imposing a minimum wage deteriorates economic performances.

The rest of the paper is structured as follows. Section 2 presents the general framework we use, which incorporates the main features of the New Economic Geography and labor market rigidities. In section 3, we solve the model in general equilibrium; we obtain a reduced form that cannot however be solved analytically because of nonlinearity. In Section 4, we thus study the impact of a rise in the domestic minimum wage on firms entry decisions using two approaches: we first derive analytically its marginal impact around a symmetric equilibrium before solving the model numerically in the general case. Last, section 5 concludes.

2 The model

The world economy is divided in two countries (or regions), Home and Foreign, with foreign variables denoted with a star. The domestic (foreign) country is populated by \( \bar{L} \)

\(^1\)Those effects are isolated thanks to the use of simplifying hypotheses. In particular, we chose to neglect the distorsive effect introduced by the unemployment system in order to make mechanisms as clear as possible. We however plan to ask for this question in future works.
(\(\tilde{L}^*\)) unskilled and \(\tilde{Q}^*\) (\(\tilde{Q}^*\)) skilled workers. Each worker offers one unit of labor to national firms\(^2\). Skilled and unskilled workers only differ by their productivity denoted \(a_Q\) and \(a_L\), with \(a_L < a_Q\). Without loss of generality we assume that productivity levels are identical across countries (\(a_L = a_L^*\) and \(a_Q = a_Q^*\)). Furthermore, we maintain the assumption of a representative agent by considering that skilled and unskilled workers are all members of the same “national family”. In each family, the representative consumer collects all labour revenues, pays taxes and then consumes.

Labor markets are perfectly competitive and define equilibrium wages, equal to the marginal revenue of each type of workers: \(w_Q = w_Q^* = a_Q\) for skilled workers and \(w_L = w_L^* = a_L\) for unskilled ones. However, the existence of a minimum legal wage implies that labor markets do not necessarily clear. If exogenous fixed minimum wages (\(\underline{w}\) and \(\overline{w}^*\)) are higher than low-skilled equilibrium wages (\(w_L < \underline{w} \leq w_Q\) and \(w_L^* < \overline{w}^* \leq w_Q^*\), as we assume in the following, some unskilled workers are let unemployed, as long as the capacity constraint is not reached. Unemployed people get unemployment benefits from an insurance system financed by lump-sum taxes on employed workers\(^3\).

Regarding the goods market, each household can consume two types of goods, a homogeneous and a differentiated good. The homogeneous good is produced under constant returns to scale in a perfectly competitive environment; it is freely traded across countries to balance the trade account. In the following, this good is taken as numeraire.

In the differentiated sector, monopolistic competing firms produce goods under increasing returns and costly trade, for both their domestic and export markets. The varieties produced by firms operating in the Home country are defined over the interval \([0 ; n]\) and indexed by \(h\). Similarly, foreign varieties are defined as \(f \in [0 ; n^*]\). The total number of varieties in equilibrium is endogenously determined, as well as firms location under free entry: firms enter a country as long as the production is profitable, given a fixed cost to produce (\(F\) units of homogenous good) and a variable cost that depends on skilled and unskilled wages. As firms operate under monopolistic competition, the number of produced varieties in equilibrium matches the number of operating firms.

\(^2\)Here, we use the standard assumption that workers are perfectly mobile across sectors but immobile internationally.

\(^3\)The assumption of a balanced unemployment insurance system financed in a lump-sum way is a strong one as it implies that the tax system has no distortive effect. Nevertheless, this symplifying hypothesis allows us to use a representative agent reasoning and isolate the influence of minimum wages on entry decisions.
2.1 Households

To preserve the representative agent assumption, we consider that all workers within a country are part of a big family that includes a representative consumer. As a result, one can derive optimal demand functions at the national level by considering the program of the representative consumer that gets the whole national income. In the following, we solve the domestic household problem; results are symmetric in the foreign country.

The utility of the representative domestic household is a positive function of her consumption of homogeneous and differentiated goods. As in Strauss-Kahn (2005), we assume the following Cobb-Douglas specification:

\[ U(C_X, C_Z) = C_X^\mu C_Z^{1-\mu} \quad 0 < \mu < 1 \]  

(1)

\( C_Z \) is the consumption level of the homogenous good \( Z \) and \( C_X \) is a composite good of all produced differentiated varieties, according to the following CES specification:

\[ C_X = (n + n^*)^{\frac{1}{1-\sigma}} \left[ \int_0^n c(h)^{\frac{\sigma-1}{\sigma}} dh + \int_0^{n^*} c(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}} \]

where \( \sigma \geq 1 \) is the elasticity of substitution across varieties.

Consumption goods are bought by the domestic household through her net of tax labor revenues and unemployment benefits. Besides, she perceives the residual profits as the owner of local firms. The representative domestic household’s income \( I \) (expressed in the numeraire good \( Z \)) thus decomposes into:

\[ I = w_Q Q + w_L L + b(\bar{L} - L) - T + \Pi \]

where \( Q \) is the total employment level of skilled workers, either in the monopolistic or the perfectly competitive sectors, and \( L \) the employment level of unskilled workers. \( (\bar{L} - L) \) is the number of unemployed unskilled workers\(^4\), that get the unemployment benefit \( b \) and \( T \) is the lump-sum transfer deducted from employed workers’ income to finance the unemployment insurance system. For this regime to be balanced, it must be true that:

\[ b(\bar{L} - L) = T \]

Last, \( \Pi \) are residual profits, equal to zero in the long-run when firms are free to enter the market.

In equilibrium, the budget constraint for the representative home household is then:

\[ \int_0^n p(h)c(h)dh + \int_0^{n^*} p(f)c(f)df + C_Z \leq Q + wL \]

(2)

\(^4\)As next section will show, the labor market for skilled workers clears in both countries.
Maximizing the representative household’s utility (1) under her budget constraint (2) leads to the optimal demand functions:

\[
C_X = \mu \frac{I}{P_X} \quad \text{(3)}
\]

\[
C_Z = (1 - \mu)I \quad \text{(4)}
\]

\[
c(h) = \left(\frac{p(h)}{P_X}\right)^{-\sigma} \frac{C_X}{n + n^*}, \quad h \in [0; n] \quad \text{(5)}
\]

\[
c(f) = \left(\frac{p(f)}{P_X}\right)^{-\sigma} \frac{C_X}{n + n^*}, \quad f \in [0; n^*] \quad \text{(6)}
\]

with \(P_X\) the expenditure-minimizing price index to purchase one unit of the basket \(C_X\):

\[
P_X = (n + n^*)^{\frac{1}{\sigma - 1}} \left[ \int_0^n p(h)^{1-\sigma} dh + \int_0^{n^*} p(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}}
\]

### 2.2 Firms

#### 2.2.1 The homogenous sector

The homogeneous good sector is perfectly competitive and integrated at the world level. Good \(Z\) is produced under constant returns to scale with a linear technology using indifferently skilled or unskilled workers\(^5\):

\[
y_Z = a_L l_Z + a_Q q_Z
\]

with \(y_Z\) the production of homogeneous good obtained from \(q_Z\) and \(l_Z\) units of skilled and unskilled labor. Since we assume that the minimum wage exceeds unskilled labor productivity, the homogeneous good sector only employs skilled workers. The profit maximization thus yields the market price for good \(Z\), denoted \(p_Z\):

\[
p_Z = \frac{w_Q}{a_Q} \quad \text{(7)}
\]

The situation in the foreign market is symmetric. As the homogenous good market is perfectly integrated, the price of good \(Z\) is equalized across countries in equilibrium. Without cross-country productivity differentials, this implies that the equilibrium wage for skilled workers is uniform at the world level.

\(^5\)This production function implies that skilled and unskilled workers are perfectly substitutable. As a consequence, this sector only employs skilled workers in an equilibrium without capacity constraints.
In the following, the homogeneous good is taken as numeraire and the skilled labor productivity is set to 1 so that, from equation (7) we get:

\[ w_Q = w_Q^* = 1 \]

The homogeneous good is used for consumption motives as well as in the differentiated good productive process to pay fixed costs. As a result, we get the resource constraint for this integrated market:

\[ y_Z + y_Z^* \geq C_Z + C_Z^* + (n + n^*)F \]

### 2.2.2 The monopolistic sector

In the monopolistic sector, total costs of production can be decomposed into fixed and variable costs. To start producing a variety \( h \), a firm incurs a fixed cost of \( F \) units of homogeneous good that implicitly defines the minimum operational profit that firms must achieve for the production to be profitable (Krugman (1991)). Once entered on the market, firm \( h \) faces a technological constraint that combines skilled and unskilled labor units according to the following CES specification:

\[ y(h) = \left[ \alpha^{-\gamma}q(h)^{\frac{\gamma}{\tau}} + (1 - \alpha)^{-\gamma}[a_Ll(h)]^{\frac{\gamma}{\tau}} \right]^{\frac{\tau}{\gamma}}, \quad \gamma > 0, \quad 0 < \alpha < 1 \]

where \( y(h) \) is the output of variety \( h \), \( q(h) \) and \( l(h) \) are skilled and unskilled labor used in the production. In this expression, \( \alpha \) is a weighting parameter that determines the share of value added paid to skilled workers whereas \( \gamma \) measures the elasticity of substitution between skilled and unskilled labor. When \( \gamma < 1 \), skilled and unskilled labor are low substitutes, while the case \( \gamma = 1 \) corresponds to a Cobb-Douglas production function and \( \gamma > 1 \) implies a high degree of substitutability between skilled and unskilled workers.

We solve the program backward by first considering the optimization problem of firms that already entered the market.

The first step consists in deriving the optimal marginal cost of production and the resulting labour demands. Minimizing the total cost function under a given production amount constraint yields the expression of the marginal cost to produce one unit of variety \( h \) (in terms of the numeraire good):

\[ MC(h) = \left[ \alpha + (1 - \alpha) \left( \frac{w}{a_L} \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \]
and the associated optimal unskilled and skilled labor demands \( l(h) \) and \( q(h) \):

\[
a_L l(h) = (1 - \alpha) \left[ \frac{w}{a_L MC(h)} \right]^{-\gamma} y(h) \quad (8)
\]

\[
q(h) = \alpha MC(h)^\gamma y(h) \quad (9)
\]

Once produced, variety \( h \) can be sold to domestic agents or exported. Shipping goods abroad entails transportation “iceberg” costs à la Samuleson (1954): to sell one unit abroad, a firm has to produce \( \tau > 1 \) units because of a real loss occurring during transport. The resource constraint for variety \( h \) is thus:

\[
y(h) \geq c(h) + \tau c^*(h)
\]

where \( c(h) \) and \( c^*(h) \) are total consumption of good \( h \) by workers from the domestic and foreign markets respectively.

Let \( p(h) \) denote the price of one unit of variety \( h \) sold in the domestic market, \( p^*(h) \) its price in the foreign market. Domestic firms total profits are then:

\[
\Pi(h) = p(h)c(h) + p^*(h)c^*(h) - w l(h) - q(h) - F \quad (10)
\]

The situation in the foreign country is perfectly symmetric. The marginal cost to produce one unit of a variety \( f \) is:

\[
MC^*(f) = \left[ \alpha + (1 - \alpha) \left( \frac{w^*}{a_L} \right)^{1-\gamma} \right]^{1/\gamma}
\]

From this, one get individual labor demands:

\[
a_L l^*(f) = (1 - \alpha) \left[ \frac{w^*}{a_L MC^*(f)} \right]^{-\gamma} y^*(f) \quad (11)
\]

\[
q^*(f) = \alpha MC^*(f)^\gamma y^*(f) \quad (12)
\]

with

\[
y^*(f) \geq c^{ast}(f) + \tau c(f)
\]

and profits of an individual firm located in the foreign market are :

\[
\Pi^*(f) = p^*(f)c^*(f) + p(f)c(f) - w^* l^*(h) - q^*(h) - F \quad (13)
\]

with \( p^*(f) \) and \( p(f) \) the market prices of a foreign variety \( f \), respectively sold in the foreign and domestic markets, \( c^{ast}(f) \) and \( c(f) \) the demand functions for good \( f \) addressed by the foreign and domestic consumers respectively.
2.2.3 Price decisions

Next step consists in determining the optimal prices set by each differentiated firm for each local and export market, given the demand functions for its good it faces. Firm \( h \) maximizes its profit (equation 10) given its optimal labour demand functions (equations 8 and 9) and the demand for good \( h \) (equation 5 and its foreign counterpart). In the monopolistic framework à la Blanchard and Kiyotaki (1987), we get that firms optimally set prices by applying a constant mark-up rate over marginal costs, multiplied by the iceberg cost in the case of export prices. Respectively for domestic and foreign varieties, equilibrium prices are then:\(^6\):

\[
\begin{align*}
 p(h) &= \frac{\sigma}{\sigma - 1} MC(h) = p \\
p^*(h) &= \tau \frac{\sigma}{\sigma - 1} MC(h) = \tau p \\
p^*(f) &= \frac{\sigma}{\sigma - 1} MC^*(f) = p^* \\
p(f) &= \tau \frac{\sigma}{\sigma - 1} MC^*(f) = \tau p^*
\end{align*}
\]

In the following, one calls \( \rho \) the relative cost to produce the differentiated good in the Home market, that only depends on the minimum wages and the unskilled labor productivity level:

\[
\rho = \frac{MC}{MC^*} = \left[ \frac{\alpha a_L^{1-\gamma} + (1 - \alpha)w_1^{1-\gamma}}{\alpha a_L^{1-\gamma} + (1 - \alpha)w^*_1^{1-\gamma}} \right]^{\frac{1}{1-\gamma}}
\]

In each country, the equilibrium price index in the increasing returns sector can then be written as:

\[
\begin{align*}
P_X &= p (n + n^*)^{\frac{1}{\sigma - 1}} \left[ n + n^* \phi p^{\sigma - 1} \right]^{\frac{1}{1-\sigma}} \equiv p (n + n^*)^{\frac{1}{\sigma - 1}} \Delta^{\frac{1}{\sigma - 1}} \\
P^*_X &= p^* (n + n^*)^{\frac{1}{\sigma - 1}} \left[ n^* \phi p^{\sigma - 1} + n^* \right]^{\frac{1}{1-\sigma}} \equiv p^* (n + n^*)^{\frac{1}{\sigma - 1}} \Delta^*^{\frac{1}{\sigma - 1}}
\end{align*}
\]

with \( \phi = \gamma^{1-\sigma} \) a parameter of “freeness” of trade that increases between 0 and 1 when trade barriers diminish or varieties become less substitutable (see Baldwin, Forslid, Martin, Ottaviano, and Robert-Nicoud (2005)).

2.2.4 Aggregate labor demands

When labor markets are not under capacity constraints (for large enough endowments), the employment level of each segment of the labor market is determined by the demand

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\(^6\)As firms in a given location are confronted with the same constraints, one can suppress indexes \( h \) and \( f \). In the following, the \( n \) varieties produced in \( H \) are distinguished from the \( n^* \) foreign varieties by the star used for foreign variables. Moreover, lower cases are used for variables concerning an individual firm whereas capital letters are used for aggregate variables.
of local firms. In the Home country, the resource constraints, respectively in the skilled
and unskilled labor market, are:

\[ Q = q_Z + \int_0^\infty q(h)dh = y_Z + \int_0^\infty \alpha MC(h)\gamma y(h)dh \] (18)

\[ L = \int_0^\infty l(h)dh = \int_0^\infty (1 - \alpha) \left( \frac{w}{a_L MC(h)} \right)^{\gamma} \frac{y(h)}{\gamma} dh \] (19)

Similar expressions hold in the foreign country.

3 The general equilibrium

3.1 Free entry and the location of the production

To characterize the model solution, we first use optimal demands and prices (derived in
sections 2.1 and 2.2.3) to rewrite profits of a domestic and a foreign firm respectively
(equations 10 and 13) as follows:

\[ \Pi(h) = \mu \left( \frac{I}{\Delta} + \phi \rho^{1-\sigma} \frac{I^*}{\Delta^*} \right) - F \] (20)

\[ \Pi^*(f) = \mu \left( \frac{I^*}{\Delta^*} + \phi \rho^{\sigma-1} \frac{I}{\Delta} \right) - F \] (21)

with:

\[ \Delta \equiv n + n^* \phi \rho^{\sigma-1} \]

\[ \Delta^* \equiv n^* + n \phi \rho^{1-\sigma} \]

Those equilibrium profit expressions put in evidence the “Home Market Effect” well-
known in the New Economic Geography literature (Baldwin, Forslid, Martin, Ottaviano,
and Robert-Nicoud (2005)): with strictly positive trade costs (\( \phi < 1 \), ceteris paribus), an
increase in domestic demand (\( dI > 0 \)) raises operating profits at Home more than abroad.
In the long-run, this induces a more than proportional share of firms to locate in the large
country in terms of demand. This effect is central in our model as it partially counteracts
the cost determinant of entry decisions, that pushes firms to locate in the relative low-cost
country.

With free entry, equilibrium operating profits are just sufficient to cover the fixed cost
of active firms and the size of each firm (i.e. the amount of an individual firm production)
is limited by the amount of fixed costs and the marginal cost:

\[ y = \frac{(\sigma - 1)F}{MC} \]  

(22)

\[ y^* = \frac{(\sigma - 1)F}{MC^*} \]  

(23)

At this point, three polar cases must be distinguished, with regards to the geographical distribution of production in equilibrium:

- two corner equilibria in which the production of differentiated good is fully concentrated in a single country (i.e. \( n = 0 \) or \( n^* = 0 \)),

- the interior equilibrium in which some varieties of the differentiated good are produced in both countries (\( n > 0 \) and \( n^* > 0 \)).

In a corner equilibrium, the number of active firms is simply determined by the corresponding zero-profit condition (equation 23 in the \( n = 0 \) case and equation 22 in the \( n^* = 0 \) case). Appendix A details the equilibrium values of \( n, n^*, I, I^*, L, L^* \) in both corner equilibria.

In the interior equilibrium however, the relative number of firms in each country is jointly determined by equations (22) and (23). Equalizing operating profits between countries, one verifies that the relative number of active firms in each country is:

\[ \frac{n}{n^*} = \frac{I(1 - \phi\rho^{\sigma-1}) - I^*\phi(\rho^{\sigma-1} - \phi)}{I^*(1 - \phi\rho^{1-\sigma}) - I\phi(\rho^{1-\sigma} - \phi)} \]  

(24)

This relation is only valid in the interior equilibrium, for \( n/n^* > 0 \), i.e. for a low enough cost gap (see appendix B for details):

\[ \frac{\phi(I + I^*)}{I^* + \phi^2 I} < \rho^{\sigma-1} < \frac{I + \phi^2 I^*}{\phi(I + I^*)} \]  

(25)

Outside this interval, production is entirely concentrated in the low-cost country. The following table summarizes the equilibrium pattern of production as a function of the relative cost to produce in each country (i.e. as a function of the relative minimum wage):

As expected, equation (24) underscores the two determinants of the spatial allocation of firms introduced in this model, namely the cost and the demand determinants. In
As explained in section 2.1, in a non-distorting unemployment insurance system, income levels solely depend on employment levels and the size of minimum wages:

\begin{align*}
I &= Q + wL \\
I^* &= Q^* + w^*L^*
\end{align*}

(26) (27)

As long as the minimum wage is lower than the equilibrium wage for skilled workers ($w \leq 1$ and $w^* \leq 1$), the skilled labor market clears in general equilibrium so that\(^7\):

\begin{align*}
Q &= \bar{Q} \quad \text{and} \quad Q^* = \bar{Q}^*
\end{align*}

As for the employment level of unskilled workers, our assumption that the minimum wage is set above unskilled workers’ productivity ($a_L < \bar{w}$) implies some positive level of unemployment. Using the optimal labor demand function (equation 19), the free-entry condition (equation 22) in the domestic country and their foreign counterparts, we get the following equilibrium nominal low-skilled employment levels:

\begin{align*}
\frac{wL}{a_LMC} &= n(1 - \alpha) \left( \frac{w}{a_LMC} \right)^{1-\gamma} (\sigma - 1)F \\
\frac{w^*L^*}{a_LMC^*} &= n^*(1 - \alpha) \left( \frac{w^*}{a_LMC^*} \right)^{1-\gamma} (\sigma - 1)F
\end{align*}

(28) (29)

\(^7\)Equilibrium is achieved through balanced international trade flows, as defined by the following balance of payments condition:

\[ n\phi \rho^{1-\alpha} \frac{\mu I^*}{\Delta^*} - n^* \phi \rho^{1-\alpha} \frac{\mu I}{\Delta} = C_Z + nF - y_Z \]
Combining equations (26) and (28) (equations 27 and 29 in the foreign case), we derive the equilibrium values for the domestic (foreign) income:

\[ I = \bar{Q} + n(1 - \alpha) \left( \frac{w}{\alpha_L} \right)^{1-\gamma} MC^{\gamma-1}(\sigma - 1)F \]  
\[ I^* = \bar{Q}^* + n^*(1 - \alpha) \left( \frac{w^*}{\alpha_L} \right)^{1-\gamma} MC^{*\gamma-1}(\sigma - 1)F \]

To complete the resolution, one needs a last relation between incomes and the number of active firms in each country. Combining the expressions of operational profits of domestic and foreign firms (equations 20 and 21) and the zero-profit conditions (equations 22 and 23) we get the following equation:

\[ (n + n^*)F = \frac{\mu}{\sigma} (I + I^*) \]  

As usual in the Dixit-Stiglitz’s framework, the total amount paid to cover fixed costs is proportional to the world expenditure in the monopolistic sector.

Equations (24), (30), (31) and (32) build a system of 4 equations in 4 unknown variables \{n, n^*, I, I^*\} that is however not tractable analytically because of the non-linearity of equilibrium relations. We thus study the analytical properties of the model by differentiating this system around a symmetric equilibrium, before simulating it numerically in the general case.

### 3.3 Symmetric equilibrium

In the symmetric equilibrium, the minimum wage and labor endowments are assumed to be identical across countries \(w = w^*, \bar{Q} = \bar{Q}^*, \bar{L} = \bar{L}^*\). As a consequence, the number of firms entering each market is the same everywhere and strictly positive, and condition (25) holds. From equations (24), (30), (31) and (32), we get:

\[ n = n^* = \frac{\mu}{\sigma - \mu(1 - \alpha)} \left( \frac{w}{\alpha_L} \right)^{1-\gamma} MC^{\gamma-1}(\sigma - 1)F \frac{\bar{Q}}{F} \]

Calling

\[ A = (\sigma - 1)(1 - \alpha) \left( \frac{w}{\alpha_L} \right)^{1-\gamma} MC^{\gamma-1} \]
one can verify that:

\[ I = I^* = \frac{\sigma}{\sigma - \mu A} \bar{Q} \]

\[ Q = Q^* = \bar{Q} \]

\[ L = L^* = \frac{\mu A}{\sigma - \mu A} \frac{\bar{Q}}{w} \]

In that case, as trade flows of differentiated goods are balanced, each country produces the quantity of homogeneous good necessary to cover the representative household’s consumption and the fixed costs paid by domestic firms:

\[ y_z = y_z^* = C_z + nF \]

4 Labor market policies and location decisions

In our framework, two unilateral labor market policies can be contemplated: an increase in unemployment benefits (\( b \)) or a change in the domestic minimum wage (\( w \)). As long as the unemployment insurance system is balanced, an increase in unemployment benefits has no impact on the general equilibrium of the model\(^8\). In this section, we therefore focus on the impact of a change in the domestic country minimum wage.

Suppose that the domestic government raises the minimum wage (\( dw > 0 \)), which affects production conditions in the differentiated good sector. The direct effect is an increase in the marginal cost to produce that tends to deter firms to enter the domestic market. On the other hand, in a non-distorsive tax system, the raise in the remuneration of unskilled workers tends to increase the global income \( I \), which can generate agglomeration in a model featured by a home market effect.

Which effect will dominate is however far from trivial. To investigate this point, we proceed in two ways. First, we focus on the impact of \( dw > 0 \) for small deviations around the symmetric equilibrium. Second, we depart from the symmetric equilibrium case by analyzing how the general equilibrium evolves with increasing values of \( w \), for a given foreign minimum wage \( \underline{w} \).

4.1 Marginal wage sensitivity of location decisions

In this section, we take as starting-point the symmetric equilibrium and analyze the marginal impact of a unilateral increase in the domestic minimum wage (\( dw > 0 \)). Analytical

\(^8\)The shock has an impact in terms of redistribution as only unskilled workers receive unemployment benefits. Under our representative household hypothesis however, one cannot ask for this effect. This feature is entirely governed by the assumption of a non-distorsive tax system.
results are obtained by differentiating the 4 equilibrium relations (24), (30), (31) and (32) around the symmetric equilibrium in the special case of a unitary substitution elasticity between skilled and unskilled workers ($\gamma = 1$). Simulations are then conducted in the general case ($\gamma \neq 1$) to confirm the previous results and highlight the role of the degree of substitutability between skilled and unskilled workers in our results.

4.1.1 With a Cobb-Douglas production function ($\gamma = 1$)

When the elasticity of substitution between skilled and unskilled labor is unitary, the production function takes the following Cobb-Douglas form:

$$y(h) = [q(h)]^\alpha[a_Ll(h)]^{1-\alpha} \quad 0 < \alpha < 1$$

The system formed by (24), (30), (31) and (32) can be rewritten as:

$$\frac{n}{n^*} = \frac{I(1 - \phi\rho^{\sigma-1}) - I^*(\rho^{\sigma-1} - \phi)}{I^*(1 - \phi\rho^{1-\sigma}) - I\phi(\rho^{1-\sigma} - \phi)}$$

$$I = \bar{Q} + n(1 - \alpha)(\sigma - 1)F$$

$$I^* = \bar{Q}^* + n^*(1 - \alpha)(\sigma - 1)F$$

$$n + n^* = \frac{\mu}{\sigma F}(I + I^*)$$

In that case, one easily verifies that the total number of active firms is exogenous in equilibrium, thus implying that $dn = -dn^*$.

Using this property when differentiating (24), we get the derivative of the number of active firms in the domestic market with respect to the relative cost to produce the differentiated good, around the symmetric equilibrium:

$$\frac{dn}{dw} = \frac{2n}{w}(1 - \alpha)\frac{\phi}{1 - \phi} - \frac{\sigma(1 - \sigma)}{\sigma(1 - \phi) - \mu(1 - \alpha)(\sigma - 1)(1 + \phi)}$$

Proposition 1 states the conditions under which a small increase in $w$ generates a relocation of firms from the foreign to the domestic country.

**Proposition 1** In the case of a Cobb-Douglas production function, an increase in the domestic minimum wage $dw > 0$ leads to an increase in the number of domestic active firms in equilibrium if and only if

$$\mu(1 - \alpha) > \frac{\sigma}{\sigma - 1}\left(\frac{1 - \phi}{1 + \phi}\right)$$

(34)
Proof. It is trivial that $\frac{\partial n}{\partial w}(1 - \alpha) - \frac{\phi}{1 - \phi} > 0$ and $\sigma(1 - \sigma) < 0$. For $\frac{dn}{dw}$ to be positive, the term $\sigma(1 - \phi) - \mu(1 - \alpha)(\sigma - 1)(1 + \phi)$ has to be negative. It is the case if condition (34) holds. ■

Proposition 1 states that, under condition (34), the demand increase in the domestic country generated by a raise in the minimum wage ($dw > 0$) is strong enough to counteract the negative impact on the domestic country’s cost competitiveness. As the home market effect dominates the cost effect, a higher share of firms is enticed to enter the market ($dn > 0$). In this case, the domestic national income benefits from the minimum wage increase (as $dI = (1 - \alpha)(\sigma - 1)Fdn$). The effect on the unskilled employment level is more complex. Consider equation 28 in difference around the symmetric equilibrium:

$$dL = \frac{L}{n}dn - \frac{L}{w}dw$$

(35)

It highlights two transmission channels through which a small increase in $w$ affects the low-skilled employment level.

- First, the direct cost effect ($dw > 0$) negatively affects labour demand for unskilled workers. *Ceteris paribus*, with an exogenous number of firms, a rise in the minimum wage would always deteriorate the employment level.

- Yet, if condition (34) holds, *i.e.* if $\frac{dn}{dw} > 0$, new firms enter the domestic market; everything else equal, it translates into a higher demand for unskilled workers. In that case, the second channel dominates the cost effect and the low-skilled employment level increases.

Condition (34) shows that the net effect of $dw > 0$ on the relative number of domestic firms depends on structural parameters. The positive impact of raising $w$ is more likely to occur when the share of differentiated good in the consumption ($\mu$) and the share of unskilled workers in the production $(1 - \alpha)$ are high, because those parameters influence the magnitude of the income effect generated by the minimum wage increase. On the other hand, the cost effect is likely to dominate for high trade costs ($\tau$) or a strong elasticity of substitution between varieties ($\sigma$) that limit the “freeness” of trade and the home market effect (see Baldwin, Forslid, Martin, Ottaviano, and Robert-Nicoud (2005)).

9Such a policy is however “neighbour-thy-neighbour” in the Cobb-Douglas case as the number of foreign firms decreases ($dn^* = -dn$) and the foreign income drops ($dI^* = -dI$).

10Note that, in this model, the size of national incomes only depends on the employment level of unskilled workers as the skilled labor market is always in equilibrium.
The degree of substitutability between skilled and unskilled labor is however likely to play a role in the final effect. We therefore investigate the role of $\gamma$ by considering the general case of a CES production function.

4.1.2 With a CES production function ($\gamma \neq 1$)

Previous results have been obtained in the case of a unitary elasticity of substitution between low-skilled and skilled workers. Yet, the empirical literature does not reach some consensus on the value for $\gamma$. In the French case for instance, Shadman-Mehta and Sneessens (1995) or Biscourp and Gianella (2001) estimate the elasticity of substitution between skill groups to be close to one. On the basis of Duguet and Gianella (1999) estimates, Salanié (2000) uses a value of 0.7, that implies a weak substitutability between skilled and unskilled workers, whereas Hamermesh (1993)’s estimate is close to 1.25.

In the CES case with $\gamma \neq 1$, the total number of produced varieties is no more independent of labor costs, and the impact of the domestic minimum wage increase on entry decisions is not symmetric across countries. As shown in appendix C, we get that:

$$\frac{dn}{dw} \neq -\frac{dn^*}{dw}, \quad \frac{dI}{dw} \neq -\frac{dI^*}{dw}$$

To further investigate the role of $\gamma$, the model is calibrated and simulated numerically. Table 1 presents the calibration values. The share of differentiated goods in the utility function $\mu$ is taken from Strauss-Kahn (2005). The value for $\alpha$ is taken from Salanié (2000)’s estimate of the share of skilled workers in the French value added during the 90s. We arbitrarily$^{11}$ set the fixed cost of production $F = 1$, $Q = 50$, the low-skilled productivity level $a_L = 0.5$ and the minimum wage $w = 0.8$. We conduct simulation exercises for two sets of values for $\tau$ and $\sigma$. Indeed, the empirical literature leaves us some room regards the values they can take, whereas proposition 1 shows that they have a key role in the responses of endogenous variables to $dw$. Regards the values for the transport cost, $\tau = 1.15$ and $\tau = 1.25$ are values within the range commonly found in the new trade literature, with $\tau$ approaching 1.15 after an integration process. The elasticity of substitution across varieties $\sigma = 3$ corresponds to the lower bound generally assumed (Venables (1996)), while $\sigma = 6$ corresponds to a mark-up rate of 25% that is usual in the macroeconomic literature (Morrison (1990)).

How does the domestic wage increase affect the location decisions when the elasticity of substitution between skilled and low skilled workers differs from one? To ask for this, $^{11}$Simulation exercises show that the values for $F, Q, a_L$ and $w$ do not play a crucial role in our results. The only requirement is $a_L < w \leq 1$. 17
we conduct a sensitivity analysis to alternative values of $\gamma$ given the calibration for the others parameters$^{12}$. Table 2 presents the results.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>Calibration 1: $\tau = 1.15$, $\sigma = 3$</th>
<th>Calibration 2: $\tau = 1.25$, $\sigma = 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\frac{dn}{dw} (\times 10^8)$, $\frac{dI}{dw} (\times 10^8)$, $\frac{dL}{dw} (\times 10^8)$</td>
<td>$\frac{dn}{dw}$, $\frac{dI}{dw}$, $\frac{dL}{dw}$</td>
</tr>
<tr>
<td>0.7</td>
<td>1.08, 1.09, 1.35</td>
<td>-46.67, -122.69, -176.91</td>
</tr>
<tr>
<td>0.9</td>
<td>1.25, 1.21, 1.49</td>
<td>-45.33, -108.25, -157.45</td>
</tr>
<tr>
<td>1.0</td>
<td>1.37, 1.29, 1.60</td>
<td>-43.30, -101.75, -148.69</td>
</tr>
<tr>
<td>1.1</td>
<td>1.53, 1.40, 1.74</td>
<td>-41.37, -95.70, -140.40</td>
</tr>
<tr>
<td>1.3</td>
<td>2.04, 1.77, 2.17</td>
<td>-37.78, -84.82, -125.48</td>
</tr>
</tbody>
</table>

Table 2: The role of gamma

As shown by columns 2 to 5, the home market effect dominates the negative cost effect in the case $\{\tau = 1.15$ and $\sigma = 3\}$: a rise in the domestic minimum wage generates an increase in the number of domestic firms, the domestic income and low-skilled employment level. Moreover, the positive impact of the labor market policy is higher the higher $\gamma$: increasing values of $\gamma$ raises the derivative $dn$ (as well as $dI$ and $dL$) to $dw$. Indeed, when $\gamma$ raises, the substitution between skilled and unskilled workers becomes easier.

- On the one hand, this mitigates the positive effect on the unskilled employment as, *ceteris paribus* (i.e. for $dn = 0$), the substitution reduces the demand for unskilled workers.

- On the other hand, it tends to reinforce the entry of firms that can produce closer to their technology frontier$^{13}$. Every thing else equal, this translates into a higher demand for both skilled and unskilled workers.

$^{12}$Furthermore we also perform sensitivity analysis to $\tau, \alpha, \mu, \sigma$ for both $\gamma < 1$ and $\gamma > 1$. Results confirm those analytically obtained in the Cobb-Douglas case. Whatever the value of $\gamma$, a small increase in the domestic minimum wage is all the more likely to have a positive impact of the number of active domestic firms ($\frac{dn}{dw}$) as $\tau$ and $\sigma$ are low and $\alpha$ and $\mu$ are strong.

$^{13}$Indeed, as skilled labor is paid to its marginal productivity while unskilled labor is paid above its marginal productivity, production becomes more efficient when firms substitute skilled workers to unskilled ones. This substitution allows firms to make more profit through a gain in competitiveness, which attracts additional firms.
In general equilibrium, despite the increase in the relative cost of low-skilled labor (to skilled labor), the home market effect combined with the positive aspect of the adjustment of factorial use entices more firms to enter the domestic market as \( \gamma \) increases and benefit to all types of workers.

Columns 6 to 9 display the sensitivity analysis results in the case \( \{\tau = 1.25, \sigma = 6\} \). Consistent with our analysis in section 4.1.1, the negative cost effect dominates in this case. Indeed, the derivatives of \( n, I, L \) to \( dw \) are always negative. They are all the more negative as skilled and low-skilled labors are low substitute \( (\gamma < 1) \). A similar interpretation holds: as the elasticity of substitution between factors increases \( (\gamma \text{ high}) \), the positive effect of the adjustment in the factorial use becomes stronger. For a given production pattern, firms are enticed to substitute skilled to unskilled workers, improving the efficiency of the production technique. The positive aspect of the adjustment of factorial use puts a brake on the decrease in unskilled labor demand, aggregate income and the number of domestic firms. The derivative of \( n, I \) and \( L \) are less negative as \( \gamma \) increases.

The analysis conducted at the neighborhood of the symmetric equilibrium thus allows to contrast situations where the cost effect of a minimum wage increase dominates, from situations with a large demand effect. It highlights some key structural parameters that influence the balance of both effects and the final result. However, the perfect symmetry is a very specific situation, that cannot be used to explain actual productive patterns in a world with a large cross-country heterogeneity in minimum wages (Dolado, Felgueroso, and Jimeno (2000)). In the following section, we thus depart from the symmetric case by relying on numerical simulations to ask for the changes in productive patterns following a sustained raise in the domestic minimum wage.

4.2 Minimum wage adjustments and the productive pattern

In the previous section we analyzed the impact of a marginal raise of \( w \) on the variation of the number of domestic, around the symmetric equilibrium. We now depart from the symmetric case by analyzing the general equilibrium results for increasing values of \( w \), given some constant value for \( w^\ast \). How does the equilibrium number of firms evolve as the relative marginal cost of low-skilled labor increases in the domestic country? What for low-skilled employment? To investigate those points, we simulate the model in the specific case of a Cobb-Douglas production function \( \gamma = 1 \), calibration displayed in table 1 and both sets of values for \( \tau \) and \( \sigma \). Figures 1 and 2 display the evolutions of the equilibrium values for \( n, n^\ast, I \) and \( L \) when \( w \) increases from 0.8 (the symmetric case) to 0.835.

As in the previous section, two situations must be distinguished: in figure 1, the cost
effect dominates and the number of firms in the domestic country is lower the higher is its minimum wage.

\begin{figure}
\centering
\begin{subfigure}{0.4\textwidth}
\centering
\begin{tikzpicture}
\begin{axis}[
    title={Number of firms},
    xlabel={$w$},
    ylabel={Number of firms},
    xmin=0.8, xmax=0.83,
    ymin=4, ymax=6
]
\addplot coordinates{
(0.8, 6)
(0.81, 5.5)
(0.82, 5)
(0.83, 4.5)
};
\end{axis}
\end{tikzpicture}
\end{subfigure}
\begin{subfigure}{0.4\textwidth}
\centering
\begin{tikzpicture}
\begin{axis}[
    title={Domestic income},
    xlabel={$w$},
    ylabel={Domestic income},
    xmin=0.8, xmax=0.83,
    ymin=60, ymax=64
]
\addplot coordinates{
(0.8, 64)
(0.81, 63)
(0.82, 62)
(0.83, 61)
};
\end{axis}
\end{tikzpicture}
\end{subfigure}
\begin{subfigure}{0.4\textwidth}
\centering
\begin{tikzpicture}
\begin{axis}[
    title={Domestic low-skilled employment level},
    xlabel={$w$},
    ylabel={Domestic low-skilled employment level},
    xmin=0.8, xmax=0.83,
    ymin=12, ymax=18
]
\addplot coordinates{
(0.8, 18)
(0.81, 16)
(0.82, 14)
(0.83, 12)
};
\end{axis}
\end{tikzpicture}
\end{subfigure}
\begin{subfigure}{0.4\textwidth}
\centering
\begin{tikzpicture}
\begin{axis}[
    title={Number of foreign firms},
    xlabel={$w$},
    ylabel={Number of foreign firms},
    xmin=0.8, xmax=0.83,
    ymin=5.5, ymax=7.5
]
\addplot coordinates{
(0.8, 7.5)
(0.81, 7)
(0.82, 6.5)
(0.83, 6)
};
\end{axis}
\end{tikzpicture}
\end{subfigure}
\caption{Sensitivity analysis, \{$\tau = 1.15, \sigma = 3$\}}
\end{figure}

In figure 2, a strong demand effect leads to a positive link between the domestic minimum wage and the number of productive firms.

Consider figure 1 (with \{$\tau = 1.25, \sigma = 6$\}). As $\underline{w}$ increases (relative to $w^*$), given that the negative cost effect dominates, the equilibrium number of domestic firms monotonically decreases, as well as the aggregate domestic income and the low-skilled employment level. Not surprisingly, the number of foreign firms increases as it decreases in the domestic country, while both countries goes on producing some varieties (in the interior equilibrium case with $n > 0$ and $n^* > 0$).

Consider now figure 2 (with \{$\tau = 1.15, \sigma = 3$\}). In this case, the strong demand effect induced by the rise in $\underline{w}$ leads to a positive link between the domestic minimum wage and the number of productive firms. As $\underline{w}$ reaches the value 0.81, the home market effect is so strong that the number of foreign firms vanishes to 0, both countries switch to a corner equilibrium with $n > 0$ and $n^* = 0$. All differentiated varieties are produced in the domestic country \textit{i.e.} the largest in terms of demand.
As \( w \) goes on increasing, the equilibrium values for \( n \) and \( I \) remain constant, while the equilibrium unskilled employment level slightly decreases. Indeed, as shown by equations 36 and 37 in appendix A, in the Cobb-Douglas case \((\gamma = 1)\), it happens that both \( n \) and \( I \) are independent of \( w \) in the corner equilibrium. On the contrary, the equilibrium level of unskilled workers is still influenced by the minimum wage level, with a negative sign. As the minimum wage goes on increasing, the higher cost of low-skilled labor (relative to \( w_Q \)) reduces the demand for unskilled workers. The employment level of unskilled workers slightly decreases, which tends to deteriorate the domestic income. In the Cobb-Douglas specification, this effect is exactly offset by the increase of \( w \), so that the equilibrium value of \( I \) remains constant\(^{14}\).

\(^{14}\)Given that in the model we always have:

\[
I = \bar{Q} + w\bar{L}
\]
5 Conclusion

Using insights of the labour market and the new economic geography literatures, our paper contributes to the living debate on the controversial of labor market policies (such as the existence of a minimum wage) on macroeconomic performances. It sheds light on an original mechanism, linked to the endogenous entry decisions of firms, through which active labor market policies can affect output and employment level. In our Dixit-Stiglitz-Krugman framework with wage rigidities and a non-distorsive unemployment insurance system, the minimum wage has a twofold effect on profits. On the one hand, by forcing firms to pay unskilled workers above their productivity, it reduces their competitiveness. On the other hand, its positive effect on the low-skilled workers’ income increases the national market potential. Using an analytical reasoning as well as numerical simulations, we are able to distinguish situations where i) the cost effect dominates, in what case a minimum wage increase deters firms to enter the market, ii) the demand effect is large enough for the wage increase to have a positive impact on the entry of firms and low-skilled employment. Moreover, the balance between those effects is shown to depend on the technology in the monopolistic sector, the form of preferences and the size of trade costs. The positive market potential effect is more likely to occur when the share of unskilled workers in the national demand is high, as the minimum wage increase only benefits to this population, and when the world is more “globalized” since it strengthens the Home Market effect.

This last result is however strongly linked to our assumption that the balanced unemployment insurance system is financed by a lump-sum tax on employed workers. This assumption allows us to derive interesting results concerning the impact of a minimum wage increase policy in a simple representative agent framework. Yet, it has necessarily a critical impact since this lump-sum tax system doesn’t have any distorsive effect on labor supply. The next step then consists in the introduction of a more sophisticated tax rate system financing the unemployment benefits. In that setting, labor supply decisions (for each type of labor) should be affected by labour market policies. An increase in the minimum wage, as well as changes in the tax rates imposed on each type of labor, are likely to alter the equilibrium unemployment level through their impact on both demand and supply of labor. As a result, the size of the home market effect is likely to be modified as well. Contrasting the results obtained in the present paper to those we would get is of particular interest and calls for future research.
References


A The general equilibrium in the corner equilibrium

A.1 The equilibrium when \( n > 0 \) and \( n^* = 0 \)

As soon as:
\[
\rho^{\sigma-1} < \frac{\phi(I + I^*)}{I^* + \phi^2 I}
\]
the relative marginal cost is so low in the domestic country that all firms are enticed to enter the domestic market to produce and serve it. The number of differentiated varieties produced in the foreign country become null. As a result, \( n^* = 0 \) while \( n > 0 \). In the foreign country, equations (29) and (27) yield the equilibrium values of \( L^* \) and \( I^* \):
\[
L^* = 0
I^* = \bar{Q}
\]
with \( Q^* = \bar{Q}^* \).

In the domestic country, we get, from equations (30) and (32), that
\[
I = \bar{Q} + n(1 - \alpha) \left[ \frac{w}{a_LMC} \right]^{1-\gamma} (\sigma - 1) \mathcal{F}
\]
\[
n = \frac{\mu \bar{Q} + I}{\sigma \mathcal{F}}
\]
Once we solve this two-equation system for the two endogenous variables \( \{I, n\} \), we determine the equilibrium value of \( L \), from equation (28):
\[
wL = n(1 - \alpha) \left[ \frac{w}{a_LMC} \right]^{1-\gamma} (\sigma - 1) \mathcal{F}
\]
while \( Q = \bar{Q} \).

A.2 The equilibrium with \( n = 0 \) and \( n^* > 0 \)

Symmetrically, as soon as
\[
\rho^{\sigma-1} > \frac{I + \phi^2 I^*}{\phi(I + I^*)}
\]
all firms are enticed to enter the foreign market to benefit from the home market effect, and the number of domestic firms reduces to 0. We thus get the following equilibrium values in both countries:
\[
n = 0
L = 0
I = \bar{Q}
\]
and

\[
\begin{align*}
n^* &= \frac{\mu \overline{Q}}{\sigma} + I^* \\
I^* &= \overline{Q}^* + n^*(1 - \alpha) \left[ \frac{w^*}{a_L^*MC^*} \right]^{1-\gamma} (\sigma - 1)F \\
\overline{w}L^* &= n^*(1 - \alpha) \left[ \frac{w^*}{a_L^*MC^*} \right]^{1-\gamma} (\sigma - 1)F
\end{align*}
\]

\section{The interior equilibrium existence condition}

In the interior equilibrium, the relative number of firms in each country is jointly determined by (22) and (23), and the relative number of active firms in each country is:

\[
n = \frac{I(1 - \phi \rho^{\sigma-1}) - I^* \phi (\rho^{\sigma-1} - \phi)}{I^*(1 - \phi \rho^{1-\sigma}) - I \phi (\rho^{1-\sigma} - \phi)}
\]

This relation is only valid in the interior equilibrium, for \( n/n^* > 0 \). It is the case if both

\[
I(1 - \phi \rho^{\sigma-1}) - I^* \phi (\rho^{\sigma-1} - \phi) > 0 \tag{38}
\]

and

\[
I^*(1 - \phi \rho^{1-\sigma}) - I \phi (\rho^{1-\sigma} - \phi) > 0 \tag{39}
\]

Manipulating equation (38) yields that:

\[
I(1 - \phi \rho^{\sigma-1}) - I^* \phi (\rho^{\sigma-1} - \phi) > 0 \quad \Leftrightarrow \quad \rho^{\sigma-1} < \frac{I + \phi I^*}{\phi(I + I^*)}
\]

Besides, after some calculus on equation 39, you get that:

\[
I^*(1 - \phi \rho^{1-\sigma}) - I \phi (\rho^{1-\sigma} - \phi) > 0 \quad \Leftrightarrow \quad \rho^{\sigma-1} > \frac{\phi(I + I^*)}{\phi^2 I + I^*}
\]

Taken together, we have that condition (24) holds if and only if condition (25) holds. For this to hold, we also have to ensure that it is always the case that

\[
\frac{\phi(I + I^*)}{\phi^2 I + I^*} < \frac{I + \phi^2 I^*}{\phi(I + I^*)}
\]
that is:

\[
[I + \phi^2 I^*][\phi^2 I + I^*] > \phi^2(+ + I^*)^2 \\
\Rightarrow I^*(1 - \phi^2)^2 > 0
\]

Provided that both aggregate incomes are positive, it it always true that \(\frac{\phi(I+I^*)}{\phi^2(I+I^*)} < \frac{I^*}{\phi(I+I^*)}\).

C Marginal wage sensitivity of location decisions in the CES case

We consider here the impact of a small increase in the domestic minimum wage \(w\) from the symmetric equilibrium case. In the general case of a CES production function, differentiating the system of equations (24), (30), (31) and (32) allows to derive analytically the derivative of the number of domestic … rms with respect to \(dw\):

\[
\frac{\partial n}{\partial w} = \frac{nA(\sigma - 1)}{w} \frac{1}{\sigma(1 - \phi) - \mu A(1 + \phi)} \\
\left[ \mu \left(1 - \frac{\mu A\phi}{\sigma - \mu A}\right) (1 - \gamma) \frac{\alpha}{1 - \alpha} \left(\frac{w}{aL}\right)^{\gamma-1} \right] \\
A - 2\sigma \frac{\phi}{1 - \phi}
\]

Moreover, as the total number of produced varieties is no more independent from labor costs in this case, the impact of the unskilled wage shock on entry decisions is not symmetric across countries:

\[
\frac{\partial n^*}{\partial w} = \frac{\mu(1 - \gamma)}{\sigma - \mu(\sigma - 1)A} \frac{n(\sigma - 1)A}{w} \frac{\partial n}{\partial w}
\]

As well, one can check that the wage shock has a differentiated impact on the income levels \(\frac{dI}{dw} \neq \frac{dI^*}{dw}\).